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RENORMALIZATION OF THE BARYON AXIAL VECTOR CURRENT IN HEAVY BARYON CHIRAL PERTURBATION THEORY AND THE LARGE N_c LIMIT

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Abstract

To understand the strong interactions in nature, the most efficient theory in agreeing with the Standard Model is the Quantum Chromodynamics (QCD) that resumes the strong interactions in a model of quarks and gluons. In the regimen of low energy, QCD has problems to describe the phenomena because here the theory is no longer a quark theory, and they are confined inside hadrons. To solve the above problem some different effective theories have been proposed, two of the most relevant theories are Heavy Baryon Chiral Perturbation theory (HBChPT), which works with a model of baryons and mesons instead of quarks, and the large-N_c limit for QCD, which considers a many colors and the expansion of any QCD operator in a spin-flavor representation. In this work, an important operator of QCD, the baryon axial vector current will be treated and computed the flavor singlet and octet contribution to its one-loop renormalization in the combined formalism of HBChPT and the large N_c limit.

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Contents

Introduction

Nature is ruled by four fundamental interactions: gravitational, electromagnetic, and strong, all of them having a description as a physical field. The first one is described by general relativity and its quantum character has two possible descriptions included in string theory and loop quantum gravity, respectively. The other interactions are included in the Standard Model, contains all the fundamental particles in three groups: leptons, quarks, and gauge bosons. Leptons are spin 1/2 particles which do not experiment strong interactions, quarks are spin 1/2 particles which are the constituents of all the hadronic matter by strong interaction and gauge bosons describe the fundamental interactions without gravity. The Standard Model does not include gravity by the hierarchy problem [1][2].

The strong interaction sector of the Standard Model is explained by Quantum Chromodynamics (QCD), which is a quantum field theory that describes the strong interactions between quarks by gluons which are the gauge bosons of the theory. Also the theory assigns a color charge as a property to each particle, analogous to the electric charge in Quantum Electrodyinamics (QED) . QCD has been proved to be a very precise and predictive theory matching with experiments, but it has two particular behaviors: confinement and asymptotic freedom, which manifest at low energy and high energy respectively. The way QCD acts, in the opposite way as QED, implies that it give us a free theory at high energies but a confined theory at low energies i.e. the quarks are confined in hadrons, so the mathematical description of the theory from the lagrangian in a quark representation will be involved [3][4].

Then, QCD have has different energetic sectors: The high energy sector, also called *perturbative regimen*, and the low energy sector that is the non-perturbative regime. The main problem in the low energy sector is which, unlike QED, it does not have a perturbative parameter to renormalize the theory, that is to say, that the theory lacks a parameter to make corrections by series expansions. For the above reasons some different frameworks to aboard the theory at low energy have been developed in the past years, for example, lattice QCD which considers the space-time as a lattice to work on it, the *large N_c limit* which promotes QCD from $N_c = 3$ to an arbitrary large N_c, where N_c is the number of colors. Another way to describe the non-perturbative regimen is to consider effective field theories, where *chiral perturbation theory* has played an important role. Particularly, *Heavy baryon chiral perturbation theory* (HBChPT) which posses a Lagrangian totally in terms of baryons and mesons and obeys a chiral symmetry, the symmetry of the massless Direc fermions. HBChPT uses the momentum as a perturbative parameter, but the main goal of the theory is that it avoids the use of quark degrees of freedom, so it works only with hadronic degrees of freedom [5][6][7].

In the last 20 years, the idea of constructing a more efficient theory mixing Heavy baryon chiral perturbation theory and the large N_c limit has been developed, to compute some baryon properties have been computed, for example masses, the magnetic moments and baryon axial vector couplings. In this work the complete one loop correction of the axial current in the mixed formalism of HBChPT and large N_c will be presented [8].

The structure of the work will be as follows. Chapters 1 and 2, a brief introduction to the framework of quantum field theory and quantum chromodynamics as well as some important tool and concepts of the theory are presented. Chapters 3 and 4 make simple construction of chiral perturbation theory and some aspects in large N_c QCD to introduce later the combined formalism. Finally, in Chapter 5 will be presented the results on the renormalization of the baryon axial vector current.

Chapter 1

Quantum Field Theory

Quantum Field Theory (QFT) arises from the idea that the photons of each energy could be treated as separate particles, then study the system as a multi-particle quantum system. This idea comes from a paper by Paul Dirac from 1927, in that sense QFT was initiated because Dirac introduced the idea of second quantization. In this chapter the main ideas about field theory will be introduced, along with some quantization mechanisms and important tools and related topics as a brief background for the discussion in the next chapters.

1.1 Quantization

In quantum mechanics, the position and momentum operators are the most commonly operators that work in an abstract Hilbert space and they have a well-known commutation relation but in general, it is not necessary to choose a particular representation. To construct the most general case it is necessary to talk about canonical operators and canonical commutation relations. First, suppose that the operator $\hat{u}$ inside a vector space U and $\hat{v}$ belongs the phase space V related by a Fourier transform with U and if the phase space is endowed with a symplectic bilinear form $w : V \times U \to \mathcal{R}$, the commutation relation between $\hat{u}$ and $\hat{v}$ takes the form:

$$\hat{u}\hat{v} - \hat{v}\hat{u} = i\hbar w(u, v). \tag{1.1}$$

Note that the election of any even-dimensional space for the phase space, has no restrictions. Now, as John von Neumann demonstrated in 1931, the operator algebra defined by (1.1) is unique within a isomorphism, and its implementations on any phase space are unitarily equivalent. Then, the chose of a system with countable infinite degrees of freedom the quantization can again be carried out with the symplectic form $w(u, v)$ on this infinite-dimensional space. This implementation is so called *second quantization* [9] and in the next section it will be explained, along with other types of quantization.

1.1.1 Canonical Quantization

The first technique to perform the quantization of a field is known as **second quantization** which is the usual way to call the canonical quantization of relativistic fields. The simplest way to understand it is by making a comparison with quantum mechanics arising from classical theory and introducing quantum notions after that. Note that first quantization refers to implementing a quantization mechanism to a mechanical system, in other words, the discrete modes of the system have been found as in the harmonic oscillator or the hydrogen atom, while the second quantization refers to the integer number of excitation of these modes.

Classical Field Theory

Classical field theory was constructed by comparison with classical mechanics and its fundamental quantity, the action S. First, in the local field theory it is possible to express the Lagrangian as an integral over the space of the *Lagrangian density* denoted by $\mathcal{L}$, for simplicity in field theory it will be called Lagrangian. This change provides us of a definition of the action by an integral in a 4-dimensional space where the Lagrangian is a function of the fields $\phi(x)$ and their derivatives $\partial_\mu \phi$, namely

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi) d^4x. \tag{1.2}$$

using the principle of least action, $\delta S = 0$, thus the field equations are

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0, \tag{1.3}$$

which are derived in a complete analogy with the Euler-Lagrange equations. Note that there is a field equation for each field ϕ. But, it is easy to assume that the construction of a Hamiltonian representation for the field theory would be simple, because it is easier to understand the canonical quantization process in this way. Then, defining the momentum density as

$$\pi(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}, \tag{1.4}$$

therefor the Hamiltonian density,here after referred to as call Hamiltonian, will be written as a Legendre transformation of the Lagrangian

$$\mathcal{H} \equiv \pi(x)\dot{\phi}(x) - \mathcal{L}. \tag{1.5}$$

As an example the Lagrangian of a non-interacting massive scalar field can be written as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2, \tag{1.6}$$

with its respective Hamiltonian

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2, \tag{1.7}$$

and its related field equation

$$\partial^\mu \phi \partial_\mu \phi + m^2 \phi = 0. \tag{1.8}$$

This is the Klein-Gordon equation, which is one of the most known field equations [1].

Second Quantization

The construction of a quantum theory, requires two new characteristics: First, the theory works with many quantum mechanical systems, each one with a different $\vec{p}$, at equal times. Second, the interpretation of the n-th excited state as a state with n particles. In QFT, the mathematical structure is based on many particle systems, then the bigger space to work is defined by the direct sum of the Hilbert spaces associated with each particle which is known as *Fock space*:

$$\mathcal{F} = \oplus_n \mathcal{H}_n, \tag{1.9}$$

and $\mathcal{H}_n$ is a Hilbert space composed by n-particle states $|p_1^\mu ... p_n^\mu\rangle$, each p_i^μ is a 4-momentum vector. Now, to understand the quantization of fields, the creation and annihilation operators need be introduced, a_p and $a_{p'}^\dagger$, which annihilates a particle with momentum p and creates a particle with momentum p' respectively. The creation and annihilation operators can be compared with the leader operators of the harmonic oscillator, which satisfy the commutation relation:

$$[a, a^\dagger] = 1. \tag{1.10}$$

In the field theory described by the Hamiltonian (1.7) one has the commutation relation:

$$[a_p, a_{p'}^\dagger] = (2\pi)^3 \delta^3(\vec{p'} - \vec{p}), \tag{1.11}$$

considering the operators located at the same time and where the constant factor is added by the Fourier transform convention taken These operators act on the Fock space states as:

$$a_{p'}^\dagger |0\rangle = \frac{1}{\sqrt{2\omega_{p'}}} |\vec{p'}\rangle. \tag{1.12}$$

Here, the state $|0\rangle$ represents a non-particle state and the constant $\omega_{p'}$ comes from the solution of the field equation (1.8):

$$p^2 = \vec{p}^- \omega_p^2 = -m^2. \tag{1.13}$$

The simplest quantum field that is possible to define is the quantum scalar field, but before introducing it, the completeness relation in terms of the momentum basis of the Fock space is needed:

$$\mathbb{1} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} |\vec{p}\rangle\langle\vec{p}| \tag{1.14}$$

then, the quantum field can be expressed in the most general form as the integral of all the possible values of the momentum for the annihilation and creation operators in the form

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left(a_p e^{ipx} + a_p^\dagger e^{-ipx} \right). \tag{1.15}$$

It is because the field equations accept a solution in a plane wave expansion and px is a 4-dimensional inner product $px = g_{\mu\nu} p^\mu x^\nu$. But this is only the time independent scalar field, if the time dependence is to be added, in Eq. (1.15) changes to

$$\phi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p(t)}} \left[a_p(t) e^{ipx} + a_p^\dagger(t) e^{-ipx} \right]. \tag{1.16}$$

Note that $a_p(t)$ and $a_p^\dagger$ are well defined at any fixed time t and (1.16) needed to change the 3-vectors by 4-vectors to include the time component [2]. This is only an example of a field operator, but there exist many types of quantum fields depending on the representation, such as vector fields, spinnorial field, etc. Our classical fields have been promoted to operators and using the same idea to promote the classical momentum to momentum operators $\pi(\vec{x},t)$, by the classical definition, it is easy to write the momentum as:

$$\pi(x) \equiv -i\int \frac{d^3p}{(2\pi)^3}\sqrt{\frac{\omega_p}{2}}\left(a_p e^{ipx} - a_p^\dagger e^{-ipx}\right). \tag{1.17}$$

To finish the quantization procedure, the next commutation relations have been obtained from the definition of the field operators

$$\begin{aligned}
[\phi(\vec{x},t),\phi(\vec{x'},t)] &= 0,\\
[\pi(\vec{x},t),\pi(\vec{x'},t)] &= 0,\\
[\phi(\vec{x},t),\pi(\vec{x'},t)] &= i\delta^3(\vec{x}-\vec{x'}),
\end{aligned} \tag{1.18}$$

finally the quantum field theory has been constructed via canonical quantization [10]. The next step will be to construct a Lagrangian derivation which will yield to the path integral formalism.

1.1.2 Path Integral

To understand the path integral mechanism, we first need to look at how it works in a quantum mechanical system. Suppose that our system has a Hamiltonian $\hat{H}$, an initial state $|i\rangle = |x_i\rangle$ and a final state $|f\rangle = |x_f\rangle$.Due to the fact that the Hamiltonian is the evolution operator an if it does not depend on time, then, the matrix element $\langle f|i\rangle$ is

$$\langle f|i\rangle = \langle x_f|e^{-i(t_f-t_i)\hat{H}}|x_i\rangle. \tag{1.19}$$

However, if $\hat{H} = \hat{H}(t)$ and it is a smooth function then it is possible to divide the interval of time in infinitesimal steps and then integrate along all the intervals. Then it has time steps in the form $t_{i+1} = t_i + \delta t$, to simplify the algebra we introduce the completeness relation of the momentum basis, namely

$$\langle x_{i+1}|e^{-iH\delta t}|x_i\rangle = \int \frac{dp}{2\pi}\langle x_{i+1}|p\rangle\langle p|e^{-i\left[\frac{\hat{p}^2}{2m}+V(\hat{x}_i,t_i)\right]\delta t}|x_i\rangle, \tag{1.20}$$

as V does not have any dependence on the momentum we can put it out of the integral and then solve using the Gaussian integral

$$\int dp\,\exp(-\frac{1}{2}ap^2 + Jp) = \sqrt{\frac{2\pi}{a}}\exp\left(\frac{J^2}{2a}\right), \tag{1.21}$$

$a = i\frac{\delta t}{m}$ and $J = i(x_{i+1} - x_i)$ which leads to

$$\langle x_{i+1}|e^{-iH\delta t}|x_i\rangle = Ne^{-iV(x_i,t_i)\delta t}e^{i\frac{m}{2}\frac{(x_{i+1}-x_i)^2}{(\delta t)^2}} = Ne^{iL(x,\dot{x})\delta t}, \tag{1.22}$$

where N is a normalization constant. Here, it has been assumed that the Lagrangian has the form $L = \frac{1}{2}m\dot{x}^2 - V(x,t)$. Finally, as Eq. (1.19) can be rewritten as the product of steps with the structure (1.22), then it reduces to

$$\langle f|i\rangle = N^n\int dx_n...dx_1 e^{iL(x_n,\dot{x}_n)\delta t}...e^{iL(x_1,\dot{x}_1)\delta t}, \tag{1.23}$$

Now, taking the limit $\delta t \to 0$ and using the definition of the action, we obtain

$$\langle f|i\rangle = N\int_{x(t_i)}^{x(t_f)} \mathcal{D}x(t)e^{iS[x]}, \tag{1.24}$$

here, $\mathcal{D}x$ means sum over all possible paths $x(t)$.

Path Integral in QFT

To understand how the path integral formalism works in QFT, let us recall that in quantum mechanics the path integral arises from the comparison between an initial and a final state, which in our case there will be an initial vacuum state at time t_i and a final vacuum state at t_f. Let us now discuss, will be discussed how the field operators ϕ and π work in a basis.

First, ϕ acts on a complete set of eigenstates as

$$\phi(x)|\Phi\rangle = \Phi(x)|\Phi\rangle, \tag{1.25}$$

where $\Phi(x)$ are eigenfunctions of ϕ and analogously for the momentum operator its respective eigenfunctions $\Pi(x)$, we have

$$\pi(x)|\Pi\rangle = \Pi(x)|\Pi\rangle, \tag{1.26}$$

these states are conjugate to $|\Phi\rangle$ and satisfy the inner product

$$\langle\Pi|\Phi\rangle = \exp\left[-i\int d^3x\Pi(x)\Phi(x)\right] \tag{1.27}$$

and the inner product between two $|\Phi\rangle$ states is

$$\langle\Phi'|\Phi\rangle = \int \mathcal{D}\Pi \exp\left(-i\int d^3x\Pi(x)[\Phi(x) - \Phi'(x)]\right), \tag{1.28}$$

also, the Hamiltonian density could be written in the form

$$\mathcal{H} = \frac{1}{2}\pi^2 + \mathcal{V}(x), \tag{1.29}$$

where $\mathcal{V}(x)$ contains all the interactions. Now, as in the quantum mechanical system, to describe how a state evolves in QFT it is necessary to divide the interval of evolution from the original state to the final state by inserting complete sets of intermediate states. In a process analogous to the quantum mechanics case, we divide $\langle 0;t_f|0;t_i\rangle$ in steps $\langle\Phi_{j+1}|e^{-i\delta tH(t_j)}|\Phi_j\rangle = N\exp(i\delta t\int d^3x\mathcal{L})$, then performing the integral over all the steps it yields

$$\langle 0;t_f|0;t_i\rangle = N\int \mathcal{D}\Phi(x)e^{iS[\Phi]}. \tag{1.30}$$

Here the action is defined as $S[\Phi] = \int d^4x\mathcal{L}[\Phi]$. Just right here, it has been considered all the possible field configurations acting on our states and it will be important in the description of Feynman diagrams.

Feynman Diagrams

In the past section, the path integral was constructed by working only with vacuum states, but, what it would happen if one or more fields were introduced in the path integral? Well, introducing only one field at a specific position and time x_j (it is a four vector but the j index is only a label) it yields

$$N\int \mathcal{D}\Phi(x)e^{iS}\Phi(x_j) = \langle 0|\phi(x_j)|0\rangle, \tag{1.31}$$

but, in general it is possible to add any number of fields

$$N\int \mathcal{D}\Phi(x)e^{iS}\Phi(x_1)...\Phi(x_n) = \langle 0|T\{\phi(x_1)...\phi(x_n)\}|0\rangle, \tag{1.32}$$

considering that on the right-hand side the operators are ordered in time. Now, as a remark, the procedures in the path integral formalism includes some types of currents, then working with them will be easier if we introduce the *generating functional* $Z[J]$ which is an usual path integral that includes a current and it represents a source in the form:

$$Z[J] = \int \mathcal{D}\phi \exp\left[iS[\phi] + i\int d^4xJ(x)\phi(x)\right]. \tag{1.33}$$

Since $Z[J]$ is a functional, the functional derivative or variational derivative will be a useful tool. It is defined in terms of the currents as

$$\frac{\delta J(x)}{\delta J(y)} = \delta^4(x-y), \tag{1.34}$$

through this definition to obtain the functional derivative for Z, we have

$$\frac{\delta Z}{\delta J(x_1)} = i\int \mathcal{D}\phi \exp\left[iS[\phi] + i\int d^4xJ(x)\phi(x)\right]\phi(x_1), \tag{1.35}$$

then,

$$-i\frac{1}{Z[0]}\frac{\delta Z}{\delta J(x_1)}\bigg|_{J=0} = \frac{\mathcal{D}\phi e^{iS[\phi]\phi(x_1)}}{\mathcal{D}\phi e^{iS[\phi]}} = \langle\Omega|\phi(x_1)|\Omega\rangle, \tag{1.36}$$

here, the $|\Omega\rangle$ state represents the vacuum state for the interacting theory while the $|0\rangle$ state represents the same in the free theory. Adding n sources

$$(-i)^n\frac{1}{Z[0]}\frac{\delta^n Z}{\delta J(x_1)\dots\delta J(x_n)}\bigg| = \langle|T\{\phi(x_1)\dots\phi(x_n)\}|\Omega\rangle. \tag{1.37}$$

For example, with the real scalar field lagrangian, we have for the free theory $Z_0[J]$:

$$Z_0[J] = \int \mathcal{D}\phi \exp\left[i\int\left(-\frac{1}{2}\phi(\Box + m^2)\phi\right) + i\int d^4xJ(x)\phi(x)\right] \tag{1.38}$$

using the well-known integral

$$\int_{\infty}^{\infty} d\vec{p}e^{-\frac{1}{2}\vec{p}A\vec{p}+\vec{J}\vec{p}} = \sqrt{\frac{(2\pi)^n}{\det A}}e^{\frac{1}{2}\vec{J}A^{-1}\vec{J}} \tag{1.39}$$

identifying the operator $i(\Box + m^2)$ as A in our formula, then it is required the construction of the inverse of our operator, this is the propagator $\Pi(x-y)$ that solves the differential equation $(\Box + m^2)\Pi(x-y) = -\delta(x-y)$, from this result follows

$$Z_0[J] = N \exp\left[\frac{1}{2}\int d^4 \int d^4 J(x)\Pi(x-y)J(y)\right], \tag{1.40}$$

therefore,

$$\begin{aligned} \langle 0|T\{\phi(x)\phi(y)\}|0\rangle &= (-i)^2 \frac{1}{Z[0]} \frac{\delta^2 Z}{\delta J(x_1)\delta J(x_2)}\bigg|_{J=0} \\ &= i\Pi(x-y), \end{aligned} \tag{1.41}$$

the Π propagator is known as the *Feynman Propagator* and since now we will denote it by D_F or simply D_{ij} for $\langle 0|T\{\phi(x_i)\phi(x_j)\}|0\rangle$ and from its differential equation, the solution is

$$\Pi(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - m^2}. \tag{1.42}$$

If we want to describe procedures in which more fields act, like in interacting theories, the theory has a more friendly representation. First, we will change de notation for the functional derivative by $\delta_j = -i\frac{\delta Z}{\delta J(x_j)}$, for the propagator $D_{xy} = \Pi(x-y)$ and we are only going to consider normalized processes. Then, introducing four fields in our process

$$\begin{aligned} \langle 0|T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\}|0\rangle &= \delta_1\delta_2\delta_3\delta_4 Z(J) \\ &= D_{34}D_{12} + D_{23}D_{14} + D_{13}D_{24}, \end{aligned} \tag{1.43}$$

This result could be represented by the diagrams in the figure 1.1.

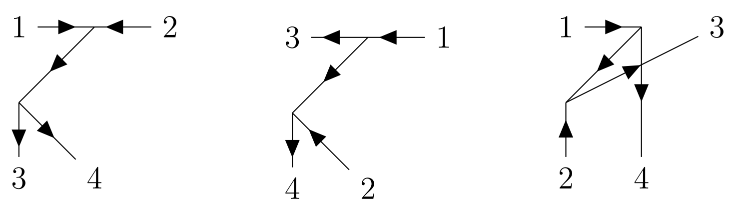


Figure 1.1: The s-channel, t-channel and u-channel respectively for the 2-2 dispersion process

These diagrams are known as *Feynman Diagrams* in the momentum-space and their mathematical interpretation is given by the *Feynman Rules*, that are well-defined in each theory[2][1].

1.2 Gauge Groups

In physics, symmetries have a great importance. To begin with let us look at how a Lagrangian changes under a infinitesimal variation $\phi \to \phi + \delta\phi$ and it is easy to check that it changes by

$$\delta\mathcal{L} = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi\right) + \frac{\delta S}{\delta\phi}\delta\phi, \tag{1.44}$$

then we define the *Noether curent* j^μ as the term in parenthesis, then

$$j^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \delta\phi \tag{1.45}$$

using this notation, and from the field equations, it implies $\delta\mathcal{L} = \partial_\mu j^\mu$, and here is the importance of the Noether currents, because a Lagrangian is invariant under a infinitesimal transformation if $\delta\mathcal{L} = 0$ i.e. if $\partial_\mu j^\mu = 0$. Then, when a Lagrangian is invariant under a transformation, it has a *continuous symmetry* and by consequence the Noether current is *conserved* [10]. Other important remarks are that the component j^0 is called charge density and $\vec{j}$ is the current density so the Noether charge is defined by

$$Q \equiv \int d^3x j^0. \tag{1.46}$$

Now, considering a finite transformation $\phi \to e^{i\alpha}\phi$ with $\alpha \in \mathbb{R}$, under this transformation the Lagriangian for a complex field $\mathcal{L} = |\partial_\mu \phi|^2 - m^2|\phi|^2$ is invariant and we usually says that the transformation is a symmetry of the Lagrangian. Additionally, when the transformation is parametrized by a scalar α we say that it is a *global symmetry*, but when the parameters are functions $\alpha(x)$ the transformation is called a *local (gauge) symmetry.*

Commonly the set of transformations, that keep invariant a Lagrangian, form a **Lie group** (see the appendix A) for example $U(1)$, $SO(3)$, etc., these groups are called *gauge group* of each theory, respectively.[2][1]

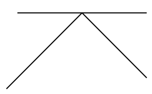
1.3 Renormalization

One of the most important things in QFT is the notion of the *renormalization* procedure that essentially we use to make predictions about long distance physics even though the theory has infinite fluctuations in short distances. To take an easy view of what renormalization is, usually, this technique appears when we want to compute an observable and we note that some divergences exist in the intermediate steps, but as we know the physical observables are finite then if is necessary to introduce a *regulator* that help us to avoid the divergences and obtain a finite result. Some examples of regulatization are hard cutoff, Paul Villars regularization, dimensional regularization, etc. and we need to make ensure that our result does not depend on the regulator which we chose.

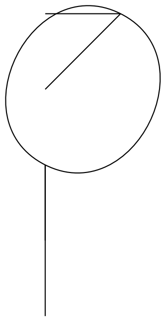
To understand renormalization an example will be presented for the massless scalar field theory with Lagrangian

$$\mathcal{L} = -\frac{1}{2}\phi\Box\phi - \frac{\lambda}{4!}\phi^4. \tag{1.47}$$

This theory is also called $\lambda\phi^4$-theory, where λ is a coupling constant in this theory. We want to compute a scattering $\phi\phi \to \phi\phi$ which at tree level is given by

$i\mathcal{M}_0 =$  $= i\lambda$ (1.48) and its 1-loop correction is

$$i\mathcal{M}_1 =$$



$$= \frac{(i\lambda)^2}{2}\int\frac{d^4}{(2\pi)^4}\frac{i}{k^2}\frac{i}{(p-k)^2}, \tag{1.49}$$

where 1/2 is a symmetry factor. As the scattering has been treated in the s-chanell we can consider $s = p^2$. As our integral is divergent we can solve it using the trick of taking a derivative respect of s, then we obtain

$$\begin{aligned}\frac{\partial}{\partial s}\mathcal{M}_1(s) &= \frac{p^\mu}{2s}\frac{\partial}{\partial p^\mu}\mathcal{M}_1(s)\\ &= \frac{i\lambda^2}{2s}\int\frac{d^4}{(2\pi)^4}\frac{1}{k^2}\frac{(p^2-p\cdot k)}{(p-k)^4},\end{aligned} \tag{1.50}$$

its solution is easily found in the literature, it reads

$$\frac{\partial}{\partial s}\mathcal{M}_1(s) = -\frac{\lambda^2}{16\pi^2 s}. \tag{1.51}$$

By taking the integral over s and choosing a suitable integration constant:

$$\mathcal{M}_1 = -\frac{\lambda^2}{32\pi^2}\ln\frac{s}{\Lambda^2} \tag{1.52}$$

and Λ is a scale parameter. Note that our result has divergences yet, but the difference between two matrix elements evaluated in two different values of s are finite and this is the real observable, then, we can chose a particular $s = s_0$ and define a *renormalized coupling* such that $\lambda_r \equiv -\mathcal{M}(s_0)$ and from the difference $\mathcal{M}(s) - \mathcal{M}(s_0)$ it is possible to obtain

$$\mathcal{M}(s) = \lambda_r - \frac{\lambda_r^2}{32\pi^2}\ln\frac{s}{s_0} + ..., \tag{1.53}$$

here it has been considered te complete correction. Finally, this is a *physical prediction* from the renormalization procedure. Another simple way to avoid the divergences is adding counterterms to the Lagrangian. For example, recovering the Lagrangian for a scalar field in the ϕ^4 theory, if we add counterterms

$$\mathcal{L} = -\frac{1}{2}\phi\Box\phi - \frac{\lambda_r}{4!}\phi^4 - \frac{\lambda_c}{4!}\phi^4, \tag{1.54}$$

where λ_c is the counterterm and computing the matrix element $\mathcal{M}$

$$\mathcal{M}(s) = -\lambda_r - \lambda_c - \frac{\lambda^2}{32\pi^2}\ln\frac{s}{\Lambda^2} + \mathcal{O}(\lambda_r^4). \tag{1.55}$$

Choosing a suitable λ_r we can recover immediately

$$\mathcal{M}(s) = \lambda_r - \frac{\lambda_r^2}{32\pi^2}\ln\frac{s}{s_0}. \tag{1.56}$$

The renormalization method using counterterms is also called *renormalized perturbation theory* and the two ways give us the same results which are really measurable quantities[2][10]. One of the most important applications of the renormalization procedure is the computation of effective coupling constants obtained from 1-loop corrections, they are also called *running couplings* and they are scale dependent i.e. they are momentum dependent. In the case of QED, this coupling is given by

$$\alpha(p^2) = e^2_{eff}(p^2) = \frac{e^2_R(p^2_0)}{1 - \frac{\beta_1}{2\pi} e^2_R(p^2_0) \ln \frac{p^2}{p^2_0}} \tag{1.57}$$

where e is the charge of the electron, e_R is the renormalized charge, which is obtained in a similar way as the renormalized coupling in the $\lambda\phi^4$-theory and e_{eff} is the effective charge. Other notable function is the β**-function** defined as

$$\beta(\alpha) \equiv \mu \frac{d\alpha}{d\mu}. \tag{1.58}$$

It has a series expansion as

$$\beta(\alpha) = -2\alpha \left[\frac{\epsilon}{2} + \frac{\alpha}{4\pi}\beta_0 + \left(\frac{\alpha}{4\pi}\right)^2 \beta_1 + \cdots \right], \tag{1.59}$$

where $\alpha = e^2_R/4\pi$ and ϵ comes from the dimensional regularization of the integrals in the minimal subtraction scheme (taking the dimensions as $d = 4 - \epsilon$). At leading order in QED with $\beta_0 = -4/3$, the β-function is

$$\beta(e_R) = -\frac{\epsilon}{2} + \frac{e^3_R}{12\pi^2}, \tag{1.60}$$

[1][10][2].

1.4 Effective Field Theory

When a physics student takes a course in classical mechanics he ignores all the relativistic or quantum effects because in the curse he considers $\hbar \to 0$ and $c \to \infty$, which is equivalent to ignore all effects outside the phenomena in this scale. That's the main idea in effective theories, as in nature exist a lot of theories that are non-renormalizable or simply they have perturbative effects what are more difficult to calculate.

In field theory the scale of the effective theory is given by distance or to be more specific by energy. Then, an effective field theory works in a given energy scale E with accuracy ϵ in terms of a finite set of parameters that satisfies:

- For each dimension we have $k - 4$ parameters
- The contribution of the interactions of dimension k is proportional to $\left(\frac{E}{M}\right)^k$.

And we can go on with this process in every scale but it is logical that in a very large scale the non-renormalizable interactions disappear and we only will have the full theory [11][7].

Chapter 2

Quantum Chromodynamics

In the Standard Model the most fundamental particle in Nature have been included: leptons (electron, muon, etc), quarks (up, down, strange, etc.) and gauge bosons (photon, gluon, etc.). In particular, Quantum Chromodynamics works on the sector of the Standard Model that describes the strong interactions between quarks by gluons, they are like photons in Quantum Electrodynamics (QED) . QCD is a non-abelian gauge quantum field theory under the symmetry $SU(3)_C$ [4]. The theory can be summarized in the next remarks.

- The Noether charge associated to the group $SU(3)_C$ is the *color charge* assigned to the quarks and gluons and transported only by the gluons. The non-abelian behavior is the responsible of the self-interaction between gluons in the theory.

- Quarks are fermions of spin 1/2,which posses both electric charge and color charge. They are grouped in two sectors: light quark sector (up, down, and strange quarks) and the heavy quark sector (charm, bottom, and top quarks). At low energies quarks are always in entangled states, also known as hadrons which are divided in two big categories: **mesons**(pion, kaon, etc.), which are bosons constructed by the union of a quark and antiquark ($q\bar{q}$) and **baryons**(proton, neutron, etc.), that are fermions formed by a triplet of quarks.

- The color charge as a quantum number emerges from the problem that represents the existence of baryon states as Δ^{++} composed of three up quarks with up spin ($u^{\uparrow}u^{\uparrow}u^{\uparrow}$) which obeys a bosonic statistics. The solution of the problem is the addition of color as a quantum number for each quark q^{β}, β =1, 2, 3 (red, blue, green) that helps to create antisymmetric baryon states. The baryon and meson states are represented by

$$B = \frac{1}{\sqrt{6}}\epsilon_{\alpha\beta\gamma}|q^{\alpha}q^{\beta}q^{\gamma}\rangle \tag{2.1}$$

$$M = \frac{1}{\sqrt{3}}\delta_{\alpha\beta}|q^{\alpha}q^{\beta}\rangle \tag{2.2}$$

- Gluons are the gauge boson associated to the strong interaction, which is the responsible of the union of the quarks in the hadrons and the transport of the color charge. Theoretically, they do not have mass or electric charge.

- From the mathematical structure and phenomenological behavior QCD possesses two important characteristics. On the one hand, *confinement*, which implies that in the low momentum regimen of the theory or in long distances the interaction becomes too strong, so quarks are always confined in hadrons. In fact applying more energy to the system to separate the quarks in a meson, for example, will be energetically more favorable the creation of a new pair rather than obtaining free quarks. On the other hand, *asymptotic freedom*, which describes that the interaction of the quarks at higher momentum or short distances is small so the quarks act like free particles[12].

2.1 The QCD Lagrangian

The development of QCD is a hard process of combined theories as the Yang Mills theory, the renormalization procedure, also adding the Feddeev-Popov terms and using the BRST symmetry to avoid extra terms that appear in the Lagrangian. The full QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}_a + \bar{\psi}_i(i\not{D} - m)\psi_i - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \mathcal{L}_{ghost} \tag{2.3}$$

where ψ_i is the fermionic field with $i = 1, ..., 6$ which includes all the quarks (six flavors and three colors), in the form

$$\psi_i = (u, d, s, c, t, b), \tag{2.4}$$

m is the mass matrix of the quarks, D is the covariant derivative defined by

$$D_\mu = \partial_\mu - igA^a_\mu T^a, \tag{2.5}$$

here μ is the space-time coordinate and runs from 1 to 4 and A^a_μ is the vector boson field of the theory, in this case the gluon with color index $a = 1, ..., 8$ and the relation between A^a_μ and A_μ is given by $A_\mu = A^a_\mu T^a$. The parameter g is the coupling constant of the strong interaction and the strength field $G^a_{\mu\nu}$ is

$$G^a_{\mu\nu} = \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^a_\nu, \tag{2.6}$$

the components of T^a are proportional to the Gell-mann matrices and f^{abc} is an antisymmetric tensor also called *structure constants* of the $SU(3)$ group, which is the gauge group of the theory. In general a $SU(N)$ group satisfies the commutation relation

$$[T^a, T^b] = if^{abc}T^c \tag{2.7}$$

and the anticommutation relation

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab}\mathbf{1} + d^{abc}T^c. \tag{2.8}$$

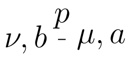
The tensor d^{abc} is symmetric and is another kind of structure constant. The structure constants are defined as

$$\begin{aligned} f^{abc} &= -2i\text{Tr}\lambda^a[\lambda^b, \lambda^c] \\ d^{abc} &= 2\text{Tr}\lambda^a\{\lambda^b, \lambda^c\}, \end{aligned} \tag{2.9}$$

these constants work in the adjoint representation of the $SU(N)$ group. The term $-\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2$ comes from the Feddeev-Popov gauge fixing but it includes some fields that do not have a physical interpretation and are called "ghosts", they are included in $\mathcal{L}_{ghost}$. The ghosts can be avoided using the *BRST symmetry*. Not only the BRST symmetry appears in the QCD Lagrangian and in the next section we will treat some of them[1][3].

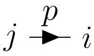
Before that, we will write the Feynman rules for QCD. For the propagators, they read:

- The gluon propagator is



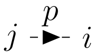
$$= i\frac{-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2}}{p^2 + i\epsilon}\delta^{ab}$$

- The propagators for colored fermions



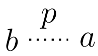
$$= \frac{i\delta^{ij}}{\not{p} - m + i\epsilon}$$

- The propagators for colored scalars



$$= \frac{i\delta^{ij}}{p^2 - M^2 + i\epsilon}$$

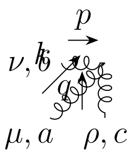
- The ghost propagator



$$= \frac{i\delta^{ij}}{p^2 + i\epsilon}$$

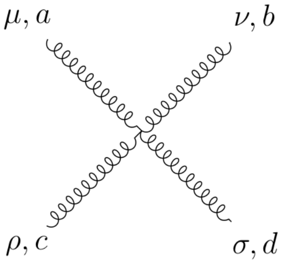
where the indices a, b are color indices and i, j are in the fundamental representation. Also, for the vertices of the theory it holds:

- The triple gluon vertex



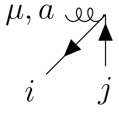
$$= gf^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$

- The four-gluon diagram



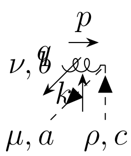
$$\begin{aligned} &= ig^2 \times [f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &\quad + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &\quad + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] \end{aligned}$$

- The interaction vertex with a fermion



$$= ig\gamma^\mu T^a_{ij}$$

- The interaction vertex for the scalar



$$= ig(k^\mu + q^\mu)T^a_{ij}$$

These are all of the Feynman Rules for QCD [2][1].

2.2 Running coupling

In Section 1.3 have been presented the running coupling for QED. In the case of QCD the β-function is

$$\beta(g_R) = -\frac{\epsilon}{2}g_R - \frac{g_R^3}{16\pi^2}, \tag{2.10}$$

here, β_0 is given by

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_F. \tag{2.11}$$

It is always positive for the well-known value $N_c = 3$ for QCD and the running coupling is

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0}\frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}}, \tag{2.12}$$

the parameter Λ_{QCD} is the scale parameter of QCD and will be useful in the next section. Note that at high momentum, the coupling constant gets weaker, this is the reason of the asymptotic freedom and in the opposite case it is responsible of the confinement[3][12][2].

2.3 Symmetries and operators

In Chapter 1 the importance of the symmetries in field theory was treated, particularly in QCD the main symmetry is given by the $SU(3)$ group. Under the symmetry a fermionic field transform as

$$\psi_i \rightarrow \psi_i + i\alpha^a T^a_{ij}\psi_j \tag{2.13}$$

and for the vector boson field

$$A^a_\mu \rightarrow A^a_\mu - f^{abc}\alpha^b A^c_\mu \tag{2.14}$$

both cases for an infinitesimal α. Using the Noether Theorem, enunciated in Chapter 1, the Noether current in this case is

$$J^a_\mu = -\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_i + f^{abc}A^b_\nu G^c_{\mu\nu}. \tag{2.15}$$

Taking only the first term we have

$$j^a_\mu = -\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_i, \tag{2.16}$$

it is also called *matter current*, is composed only by fermions, and satisfies $D_\mu j^a_\mu = 0$. Now, we need to see clearer the gauge groups, first in QCD we have a gauge group $SU(N_c)$ were N_c is the number of colors, by experimental results it was fixed to $N_c = 3$. But, the color symmetry is not the unique symmetry in QCD, now we also have the flavor symmetry that implies the gauge group $SU(N_f)$ where N_f is the number of flavors. This symmetry could be decomposed in the *chiral symmetry*, which is defined by the group $SU(N_f)_L \times SU(N_f)_R$ and we will describe bellow [10][1].

2.3.1 Chiral Symmetry

The quark sector of the QCD lagrangian is purely fermionic, it could be written as a combination of left handed and right handed spinors through the $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4$ matrix, which satisfies $\{\gamma^\mu, \gamma_5\} = 0$ and $\gamma_5^2 = 1$. Taking the transformations for the usual quark spinors ψ as

$$\psi_R = P_R\psi, \qquad \psi_L = P_L\psi \tag{2.17}$$

where

$$P_R = \frac{1+\gamma_5}{2}, \qquad P_L = \frac{1-\gamma_5}{2}, \tag{2.18}$$

it is then possible to take the Lagrangian to its chiral representation

$$\mathcal{L}_{QCD} = \bar{\psi}_{R,i} \not{D} \psi_R + \bar{\psi}_{L,i} \not{D} \psi_{L,i} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}_a, \tag{2.19}$$

where for simplicity have been omitted the gauge and ghost terms. This Lagrangian obviously is invariant under chiral transformations because we are only working in the light quark sector (u, d, s), so it is possible to suppress the mass term in the Lagrangian or it can be included taking the limit $m_u = m_d = m_s$. In general, as quarks do not have the same mass the theory would not be invariant under the chiral symmetry, for this reason this symmetry is not an exact symmetry and is called an *approximate symmetry*[13]. Now, considering the flavor symmetry in the usual QCD Lagrangian and using the γ_5 matrix, the analogous of the chiral transformation can be built. These transformations are called axial flavor transformations. The infinitesimal transformations zre written as

$$\psi_i \to \psi_i + i\alpha^a T^a_{ij}\gamma_5\psi_j, \tag{2.20}$$
$$\bar{\psi}_i \to \psi_i + i\alpha^a \bar{\psi}\gamma_5 T^a_{ij}. \tag{2.21}$$

for massless quarks, the QCD lagrangian exhibits two conserved currents

$$\mathcal{V}^a_\mu = \bar{\psi}_i\gamma^\mu T^a_{ij}\psi_i, \tag{2.22}$$
$$\mathcal{A}^a_\mu = \bar{\psi}_i\gamma^\mu\gamma_5 T^a_{ij}\psi_i, \tag{2.23}$$

and their respective Noether charges

$$V^a = \int d^4x \mathcal{V}^a_0, \qquad A^a = \int d^3x \mathcal{A}^a_0. \tag{2.24}$$

These charges transform, under the flavor $[SU(N_f)]$ transformations and the axial flavor transformation, and satisfy the algebra

$$[V^a, V^b] = if^{abc}V^c, \tag{2.25}$$
$$[A^a, A^b] = if^{abc}V^c, \tag{2.26}$$
$$[V^a, A^b] = if^{abc}A^c. \tag{2.27}$$

This algebra can be written to obtain simplified and more suitable results. For this purpose, let us define

$$F^a_L = \frac{1}{2}(V^a - A^a), \qquad F^a_R = \frac{1}{2}(V^a + A^a) \tag{2.28}$$

and, after applying the algebra one obtains

$$[F_L^a, F_L^b] = i f^{abc} F_L^c, \tag{2.29}$$
$$[F_R^a, F_R^b] = i f^{abc} F_R^c, \tag{2.30}$$
$$[F_L^a, F_R^b] = 0. \tag{2.31}$$

The most important result of expressing the vector and axial charges as left-handed and right-handed charges is that the latter operate separately and decoupled, each one generates its respective $SU(N_f)$ group of transformations, then the axial-flavor symmetry could be represented by the tensorial product of both as $SU(N_f)_L \times SU(N_f)_R$ also called *chiral symmetry*, and flavor symmetry forms a subgroup $SU(N_f)_V$, this is the flavor group [14].

2.4 Spontaneous Symmetry Breaking

As it was described in the above section, the continuous symmetries in the Lagrangian yield conserved currents and conserved charges via the Noether Theorem and the conserved charges Q generate symmetry transformations. For definition, a conserved charge satisfies the relation $[H, Q] = 0$ and even though the free Lagrangian and the potential of the theory are invariant under the symmetry transformation, the vacuum state (ground state of QFT) would no longer be. For example, in the Mexican hat potential two possibilities exist. The first one that is a symmetric vacuum in the top of the hat that satisfies $Q|\Omega\rangle = 0$ but it's unstable and the other possibility is the charged vacuum on the bottom of the hat, that is stable but the charge does not annihilate the vacuum $Q|\Omega\rangle \neq 0$. The main consequence of have a charged vacuum is the degeneracy of the vacuum, since $H|\Omega\rangle = E_0|\Omega\rangle$, applying the same Hamiltonian to the $Q|\Omega\rangle$ state

$$HQ|\Omega\rangle = [H, Q]|\Omega\rangle + QH|\Omega\rangle = E_0 Q|\Omega\rangle, \tag{2.32}$$

therefore, the ground state is degenerate. This phenomenon is known as **spontaneous symmetry breaking**. From the degenerated vacuum. the construction of momentum states, evolving the vacuum, follows as

$$|\pi(\vec{p})\rangle = C \int d^3x e^{i\vec{p}\cdot\vec{x}} J_0(x)|\Omega\rangle. \tag{2.33}$$

These states have energy $E(\vec{p}) + E_0$ and C is a suitable integration constant. Since the states are subject to the initial condition $|\pi(0)\rangle = CQ|\Omega\rangle$, then the energy $E(\vec{p})$ goes to zero when the momentum goes to zero. Thus, the $|\pi\rangle$ states must be massless states. The *Goldstone's Theorem* is a consequence of the last result, it implies the existence of massless particles from the spontaneous breaking of continuous global symmetries; these massless particle states $|\pi\rangle$ are also called *Goldstone bosons*. In the last section, it was introduced the chiral symmetry, is possible to construct an effective Lagrangian based on this symmetry called the *chiral Lagrangian* and from the spontaneous symmetry breaking of chiral symmetry in this case we will obtain the octet of Goldstone bosons which will correspond to be the meson octet [2][1][13].

Chapter 3

Effective Field Theories in QCD

In the previous chapter we could see that QCD is the quantum gauge field theory which describes the strong interactions in the Standard Model, but it has some important problems and the main of them is that his non-abelian behavior which made it a non-renormalizable theory, i.e. the divergences in the theory are impossible to avoid using the usual techniques that were developed in QED or in other renormalizable models. Thus, one of the obvious solution of this problem is the use of Effective Field Theories, that have been treated in the first chapter.

Recalling from the first chapter, an effective theory is a useful tool that gives us a good description of a theory in a particular scale of energy and momentum in this case that scale is Λ_{QCD}. This scale parameter make a constraint in our degrees of freedom, taking into account only the particles that are smaller than Λ_{QCD}, and the quantities that have the same order or higher than our scale parameter will be absorbed by the coupling constant of the Lagrangian. A non-perturbative theory is a low-energy theory that help us to obtain a good description of the phenomena. In QCD we use the scale parameter Λ_{QCD} to treat our quantities making an expansion around Λ_{QCD}, in this way we can create a controlled expansion of operators in power series of $1/\Lambda_{QCD}$ to describe an observable and working out as an effective theory we can stop our expansion in a particular order in $1/\Lambda_{QCD}$ and only a finite number of operators in the expansion that contributes to the physical phenomena. In the Lagrangian, the last ideas look as a finite number of counterterms that provide us with an effective Lagrangian with predictive power but the results have a finite precision defined by our previous expansion [7].

3.1 Heavy Baryon Chiral Perturbation Theory

The effective Lagrangian that we choose in QCD has the previously mentioned chiral symmetry related to the $SU(3)_L \otimes SU(3)_R$ group, considering the flavors of quarks, but this symmetry is spontaneously broken to the diagonal $SU(3)$ subgroup. Because of this, the effective Lagrangian obtains a modification by adding the pseudoscalar octet of pseudo-Goldstone bosons, they are the pion fields. Additionally, as Georgi postulated the heavy quark formalism [15] and in the same way Jenkins and Manohar developed the heavy baryon theory [5][16], we can treat the baryon fiels as heavy static fermions. It can be useful and will acquire more physical sense, considering that the momentum transfer from the gluons to the hadrons is minimal in comparison with the momentum transfer of the heavy baryons and as other consideration, the baryonic field depends on the velocity [17]. The previous postulates are know as *Heavy Baryon Chiral Pertubation theory* (HBChPT), and to begin with the description we will introduce the chiral Lagrangian of its lowest order, including contributions of the octect and decuplet[17]:

$$\begin{aligned}\mathcal{L}_{baryon} &= i\mathrm{Tr}\bar{B}_v(v\cdot\mathcal{D})B_v - i\bar{T}^\mu_v(v\cdot\mathcal{D})T_{v\mu} + \Delta\bar{T}^\mu_v T_{v\mu} + 2D\mathrm{Tr}\bar{B}_v S^\mu_v\{\mathcal{A}_\mu, B_v\} \\ &+ 2F\mathrm{Tr}\bar{B}_v S^\mu_v[\mathcal{A}_\mu, B_v] + \mathcal{C}(\bar{T}^\mu_v\mathcal{A}_\mu B_v + \bar{B}_v\mathcal{A}_\mu T^\mu_v) + 2\mathcal{H}\bar{T}^\mu_v S^\nu_v\mathcal{A}_\nu T_{v\mu}\end{aligned} \tag{3.1}$$

where D, F, $\mathcal{C}$ y $\mathcal{H}$ are the pion-baryon couplings, also known as chiral coefficients, Δ is the mass spliting of th octet and decuplet of baryons given by $\Delta = M_T - M_B$, B_v and T^μ_v are the baryon fields with velocity v^μ. In the effective Lagrangian appears also the matrix of pseudo-Goldstone bosons given by $\xi = exp(i\Pi/f)$, where

$$\Pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \tag{3.2}$$

and $f \approx 93$ MeV is the the pion decay constant. The combination of thr pion fields which transforms as a vector V^μ and as an axial-vector A^μ

$$V^\mu = \frac{1}{2}\left(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi\right), \qquad A^\mu = \frac{i}{2}\left(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi\right) \tag{3.3}$$

will be useful to write down the lagrangian. The baryon octet is given by

$$B = \begin{pmatrix} \Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \tag{3.4}$$

and its adjoint

$$\bar{B} = \begin{pmatrix} \bar{\Sigma}^0 + \frac{1}{\sqrt{6}}\bar{\Lambda} & \bar{\Sigma}^+ & \bar{\Xi}^- \\ \bar{\Sigma}^+ & -\frac{1}{\sqrt{2}}\bar{\Sigma}^0 + \frac{1}{\sqrt{6}}\bar{\Lambda} & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & -\frac{2}{\sqrt{6}}\bar{\Lambda} \end{pmatrix}, \tag{3.5}$$

these fields transform under chiral symmetry as

$$B_v \to UB_vU^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{3.6}$$

the decuplet of baryons with spin 3/2 is a Rarita-Schwinger T^μ field that satisfies $\not{T} = 0$ and is symmetric in its flavor indices. The explicit decuplet is

$$\begin{aligned} T_{uuu} &= \Delta^{++} & T_{uud} &= \tfrac{1}{\sqrt{3}}\Delta^+ & T_{udd} &= \tfrac{1}{\sqrt{3}}\Delta^0 \\ T_{ddd} &= \Delta^- & T_{uus} &= \tfrac{1}{\sqrt{3}}\Sigma^{*+} & T_{uds} &= \tfrac{1}{\sqrt{6}}\Sigma^{*0} \\ T_{dds} &= \tfrac{1}{\sqrt{3}}\Sigma^{*-} & T_{uss} &= \tfrac{1}{\sqrt{3}}\Xi^{*0} & T_{dss} &= \tfrac{1}{\sqrt{3}}\Xi^{*-} \\ T_{sss} &= \Omega^-, \end{aligned} \tag{3.7}$$

as these fields have velocity, their dependence is given by

$$B_v(x) = \exp(iM_B\not{v}_\mu x^\mu)B(x), \qquad T^\mu_v(x) = \exp(iM_T\not{v}_\mu x^\mu)B(x) \tag{3.8}$$

and finally we have the spin operator

$$S^\mu_v = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu, \tag{3.9}$$

and in this theory the covariant derivative is

$$\mathcal{D}^{\mu} = \partial^{\mu} + [V^{\mu}, \cdot] \,. \tag{3.10}$$

As in the usual way, now we need to compute the Green's functions from the effective Lagrangian. The propagators are

$$\frac{i}{k \cdot v} \tag{3.11}$$

for the octet and

$$\frac{iP_v^{\mu\nu}}{(k \cdot v - \Delta)} \tag{3.12}$$

for the decuplet; note that this propagator contains a polarization projector defined by

$$P_v^{\mu\nu} = (v^{\mu}v^{\nu} - g^{\mu\nu}) - \frac{4}{3}S_v^{\mu}S_v^{\nu}. \tag{3.13}$$

In the propagators we have added a momentum k which is the amount by which the baryon is off-shell and the full momentum of the baryon is

$$p^{\mu} = M_B v^{\mu} + k^{\mu}. \tag{3.14}$$

Now, in HBChPT we can define an $SU(6)_v$ symmetry transformation that transforms B_v to T_v with the same velocity, then through this symmetry it is possible to obtain relations for the couplings

$$F = \frac{2}{3}D, \qquad \mathcal{C} = -2D, \qquad \mathcal{H} = -3D \tag{3.15}$$

The renormalization of the baryon acial vector current will be discussed as an example of how the theory works [5][16][17][18].

3.1.1 Baryon axial current in HBChPT

Following the usual Noether procedure we can easily construct the baryon axial vector currens in the SU(3) symmetry limit at the lowest order from the contributions of the octet and decuplet and the result is [19]

$$\begin{aligned} J_{\mu}^{A} = {} & D\mathrm{Tr}\bar{B}_v S_u^{\mu}\{\xi T^A \xi^{\dagger} + \xi^{\dagger} T^A \xi, B_v\} + F\mathrm{Tr}\bar{B}_v S_u^{\mu}[\xi T^A \xi^{\dagger} + \xi^{\dagger} T^A \xi, B_v] \\ & + \frac{1}{2}v^{\mu}\mathrm{Tr}\bar{B}_v S_u^{\mu}[\xi T^A \xi^{\dagger} - \xi^{\dagger} T^A \xi, B_v] + \frac{1}{2}v^{\mu}\bar{T}_v^{\nu}\left(\xi T^a \xi^{\dagger} - \xi^{\dagger} T^A \xi\right) T_{v\nu} \\ & + \mathcal{H}\bar{T}_v^{\nu} S_v^{\mu}\left(\xi T^a \xi^{\dagger} - \xi^{\dagger} T^A \xi\right) T_{v\nu} + \frac{1}{2}\mathcal{C}\bar{T}_{v\mu}\left(\xi T^a \xi^{\dagger} + \xi^{\dagger} T^A \xi\right) B_v \\ & + \frac{1}{2}\mathcal{C}\bar{B}_v\left(\xi T^a \xi^{\dagger} + \xi^{\dagger} T^A \xi\right) T_{v\mu}. \end{aligned} \tag{3.16}$$

and its one loop organized can be written as

$$\langle B_i | J_{\mu}^{A} | B_j \rangle = \left[\alpha_{ij}^{A} + \left(\bar{\beta}_{ij}^{a} - \bar{\lambda}_{ij}\alpha_{ij}^{A}\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(\frac{M_K^2}{\mu^2}\right)\right]\bar{u}_{B_i}\gamma_{\mu}\gamma_5 u_{B_j} \tag{3.17}$$

where M_K is the kaon mass, α_{ij}^{A} are the tree-level contributions, $\bar{\lambda}_{ij}$ are the one-loop corrections from the wave function renormalization and $\bar{\beta}_{ij}^{A}$ are the correction due all the other graphs. In

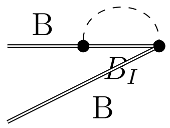


Figure 3.1: Meson loop renormalization for the baryon wave function

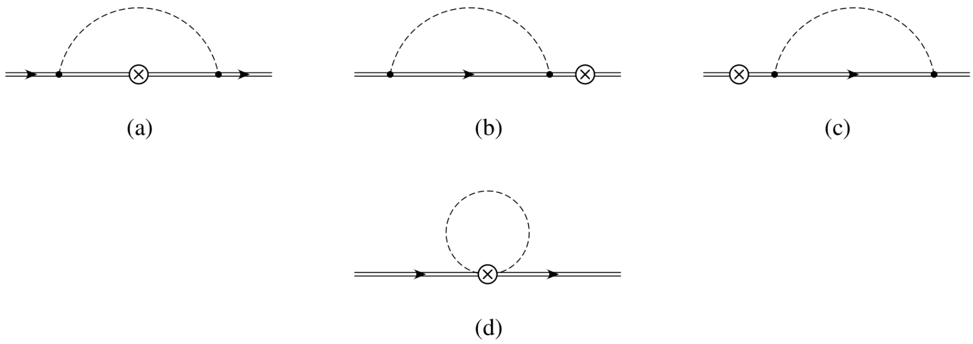


Figure 3.2: loops

this notation $\bar{\alpha} = \alpha + \alpha'$, α is the contribution from diagrams with intermediate octet lines, a' is the contribution from diagrams with intermediate decuplet lines and $\bar{\alpha}$ includes all the contributions.

All the α^A_{ij}, $\bar{\lambda}_{ij}$ and $\bar{\beta}^A_{ij}$ can be computed in terms of the chiral coefficients D, F, $\mathcal{C}$ and . The dependence of the matrix elements of the axial current on the scale parameter μ is due to the fact that (3.17) only contains the leading non analytic correction of the form $m_s \ln m_s$, where m_s is the strange quark mass, but also have M_K^2 terms with coefficients which come from operators with higher dimension in the Lagrangian. So, the μ dependence cancels the anomalous dimensions of these coefficients [19]. In a more general way, the renormalized axial current can be written as [12]

$$\langle B_i | J^A_\mu | B_j \rangle = \left[\alpha_{ij} - \sum_\Pi \left(\bar{\beta}^\Pi_{ij} - \bar{\lambda}^\Pi_{ij} \alpha_{ij} \right) F(m_\Pi, \Delta, \mu) + \sum_\Pi \gamma^\Pi_{B_i B_j} I(m_\Pi, \mu) \right] \times \bar{u}_{B_i} \gamma_\mu \gamma_5 u_{B_j} \tag{3.18}$$

where $F(m_\Pi, \Delta, \mu)$ is the one loop integral

$$\delta^{ij} F(m_\Pi, \Delta, \mu) = \frac{i}{f^2} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 k (\mathbf{k}^i)(-\mathbf{k}^j)}{(k^2 - m_\Pi^2)(k \cdot v - \Delta + i\epsilon)}, \tag{3.19}$$

This integral has been solved by Jenkins and Manohar in [19] and μ is the scale parameter from the dimensional regularization. The integral comes from the one loop renormalization of the baryon wave function given by Fig. 3.1 in the graph the double line represents a baryon, the dashed line represents a meson and their intersections $\bullet$ are strong interaction vertices. To compute the loop correction is needed to take the sum over all the possible intermediate baryons B_I. Then, the tree level contributions are included in α_{ij}, the $\bar{\beta}_{ij}$ factors are the one loop corrections given by (a) in the diagram in Fig. 3.2 where the insertion $\otimes$ is an axial vector current insertion. The coefficients $\bar{\lambda}^\Pi_{ij}$ are the corrections due to the one loop renormalization of the wave function by (b) and (c) and the $\gamma^\Pi_{B_i B_j}$ coefficients comes from the diagram (d). The integral $I(m_\Pi, \mu)$ is solved using dimensional regularization as [20]

$$I(m_\Pi, \mu) = \frac{i}{f^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_\Pi^2} = \frac{m_\Pi^2}{16\pi^2 f^2}\left[\ln\left(\frac{m_\Pi^2}{\mu^2}\right) - 1\right], \tag{3.20}$$

note that taking only the limit $m_u = m_d = 0$ and using th Gell-Mann-Okubo mass formula we can rewrite $m_\eta^2 = \frac{4}{3}m_K^2$ and recover the Eq. (3.2) [17][21].

3.2 Large-N_c QCD

The large-N_c limit is an effective field theory that provides us with an approach to analyze the behavior of QCD in a non-perturbative regimen. One problem in QCD is the absence of a small expansion parameter that help us to compute low-energy properties of hadrons, but it changed since t'Hooft realized that this parameter could be the number of colors N_c and that the theory becomes simpler when the limit $N_c \to \infty$ is taken as we can expand the QCD operators in a $1/N_c$ expansion[22]. In the large-N_c limit the baryon sector of QCD has the contracted symmetry $SU(2N_F) \supset SU(N_F) \times SU(2)$, where N_F is the number of colors. This symmetry is also known as the *spin-flavor symmetry*, arises from consistence conditions on meson-baryon scattering amplitudes and unitarity. From the large-N_c limit it is possible to get result for baryon axia vector currents and magnetic moments up to order $\mathcal{O}(1/N_c^2)$ and for baryon masses up to order $\mathcal{O}(1/N_c^3)$ using two and three light-quark flavors. The success of the large-N_c limit is that their predictions are in good agreement with experiment and in the explanation of several phenomena in the baryon sector of QCD[19].

3.2.1 Spin-flavor symmetry for large N_c baryons

The most important advantage of the spin flavor symmetry in the large N_c theory is that it provides us with a organizational framework for the $1/N_c$ expansion. The way in which the expansion works is taking the static baryon matrix element of the 1-body operators in QCD, for example the axial current, and expand it in the form [8]

$$\mathcal{O}_{QCD}^{1-body} = \sum_n \frac{1}{N_c^{n-1}}\mathcal{O}_n, \tag{3.21}$$

where the $\mathcal{O}_n$ are the independent operator polynomials of degree n in terms of the baryon spin flavor generators. In particular, it is possible to construct a basis for the $1/N_c$ expansion in terms of $\mathcal{O}_n$ and they have a natural interpretation as n body operators that act on the spin and flavor indices over the n static quarks, but to understand these operators first we need the to have a quark representation. Now, in the large N_c formalism the quark representation of the spin-flavor symmetry is based on the non-relativistic quark model picture that is useful because it give us a convenient way to write the results of the $1/N_c$ expansion. In the quark representation we have operators for creation $q_{j\beta}^\dagger$ and annihilation $q^{i\alpha}$ that satisfy the commutation relation:

$$[q^{i\alpha}, q_{j\beta}^\dagger] = \delta_j^i \delta_\beta^\alpha \tag{3.22}$$

where the indices $i, j = 1, 2, 3$ are spin indices and $\alpha, \beta = 1, .., N_F$ are flavor indices, the n-body quark operators can be constructed using these operators. Now, using the spin flavor symmetry we will construct a classification of n-body quark operators. First, the unique 0-body operator is the identity $\mathbb{1}$, which only acts on baryons, not on quarks. The 1-body operators that act on a single quark are

$$\begin{aligned} J^i &= q^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1}\right) q, \\ T^a &= q^\dagger \left(\mathbb{1} \otimes \frac{\lambda^a}{2}\right) q, \\ G^{ia} &= q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2}\right) q, \end{aligned} \tag{3.23}$$

where σ^i are the Pauli matrices corresponding to the $SU(2)$ group with $i = 1, 2, 3$ for spin and λ^a are the generators of $SU(N_F)$ with $a = 1, ..., N_F^2 - 1$, which in the particular case of $N_F = 3$ are the Gell-Mann matrices with $a = 1, ..., 8$. To apply these operators in a baryon we need to sum over al the N_c quark lines in the baryon, so in this way we can construct a number operator as

$$q^\dagger q = \sum_{l=1}^{Nc} q_l^\dagger q = N_c \mathbb{1}. \tag{3.24}$$

In general, the n-body operators with $n \geq 2$ can be written as completely symmetric product of operators of degree n in the baryon spin-flavor generators. For example, a 2-body operator can be written as

$$J^i T^a = \sum_{l,l'} \left(q_l^\dagger \frac{\sigma}{2} q_l\right) \left(q_{l'}^\dagger \frac{\lambda^a}{2} q_{l'}\right). \tag{3.25}$$

Finally, as J^i, T^a and G^{ia} are the generators of the $SU(2N_F)$ group, they satisfy a well defined algebra, given by the relations

$$\begin{aligned} [J^i, J^j] = i\epsilon^{ijk} J^k, \quad [T^a, T^b] = i f^{abc} T^c, \quad [J^i, T^a] = 0 \\ [J^i, G^{ja}] = i\epsilon^{ijk} G^{ka}, \qquad [T^a, G^{ib}] = i f^{abc} G^{ic}, \\ [G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2N_F}\delta^{ab}\epsilon^{ijk} J^k + \frac{i}{2}\epsilon^{ijk} d^{abc} G^{kc}, \end{aligned} \tag{3.26}$$

where ϵ^{ijk} is the fully antisymmetric Levi-Civita tensor, f^{abc} and d^{abc} are the structure constants of the $SU(2N_F)$ group [23][24][19].

3.2.2 Properties of baryons

In this section we will construct some static properties of baryons using the $1/N_c$ expansion, also considering the first order in symmetry breaking of $SU(3)$. By this way is possible to construct baryon axial-vector currents, baryon masses, magnetic moments, meson couplings and hyperon nonleptonic decay amplitudes [25][20], for example.

Axial vector current

The axial vector current A^{ia} transforms as $(1, adj)$. Note that we only use the space components of the axial vector current because they are the ones which that have nonzero matrix elements at zero recoil, or equivalently, the time component has been suppressed by the large N_c limit. Now, the $1/N_c$ expansion for A^{ia} can only contain a single flavor contribution (G^{ia} or T^a) by the use of the reduction rules. So the only 1-body operator is G^{ia} and for the two-body operators we have

$$\begin{aligned} \mathcal{O}_2^{ia} = \epsilon^{ijk}\{J^j, G^{ka}\} = i[J^2, G^{ia}], \\ \mathcal{D}_2^{ia} = J^i T^a. \end{aligned} \tag{3.27}$$

For the three-body case

$$\begin{aligned} \mathcal{O}_3^{ia} = \{J^2, G^{ia}\} - \frac{1}{2}\{J^i, \{J^j, G^{ja}\}\}, \\ \mathcal{D}_3^{ia} = \{J^i, \{J^j, G^{ja}\}\}, \end{aligned} \tag{3.28}$$

after this, we can apply the recursive formulas using J^2 to create n-body operators as

$$\begin{aligned} \mathcal{O}_{n+2}^{ia} = \{J^2, \mathcal{O}_n^{ia}\}, \\ \mathcal{D}_{n+2}^{ia} = \{J^2, \mathcal{D}_n^{ia}\}. \end{aligned} \tag{3.29}$$

The operators $\mathcal{D}_n^{ia}$ are diagonal operators because only have nonzero matrix elements between equal spin states while the operators $\mathcal{O}_n^{ia}$ are purely off diagonal, so only connect states with different spin. As a remarkable fact G^{ia}, $\mathcal{D}_n^{ia}$ and $\mathcal{O}_n^{ia}$ form a complete set of linear independent spin-1 adjoint operators. Then, the $1/N_c$ expansion of A^{ia} is [20]

$$A^{ia} = a_1 G^{ia} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{ia} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{ia}, \tag{3.30}$$

the coefficients a_1, b_n and c_n have $1/N_c$ expansions beginning at order are order unitary. [26][27][6][23][20].

Chapter 4

Heavy baryon chiral perturbation theory in the large-N_c limit

In the way to construct a more efficient effective field theory with better results, a notable idea is to combine the two formalism of heavy baryon chiral perturbation theory and the large-N_c limit to develop a more efficient theory. Two successes of the combined formalism are the construction of a $1/N_c$ chiral Lagrangian for the lowest-lying baryons and the renormalization of the baryon axial vector current[20]. In this chapter, the combined formalism, the Lagrangian of the combined formalism and the techniques for the renormalization of the axial current will be presented.

4.1 $1/N_c$ baryon chiral Lagrangian

To construct the $1/N_c$ baryon chiral Lagrangian we will use the previous structures of the pion nonet, the spin-1/2 octet of baryons and the spin-3/2 decuplet of baryons, that have been presented for the chiral Lagrangian for baryons and as we know, any QCD operator under the $SU(2N_F)$ has an expansion in terms of n-body operator, so the chiral Lagrangian has an expansion considering the large N_c limit[27][28][29]. To formulate the Lagrangian we will consider the baryons as heavy static fields with fixed velocity v^μ as in the chiral Lagrangian. Now, the $1/N_c$ chiral Lagrangian possesses a new symmetry $SU(2) \times U(3)$ at leading order in the $1/N_c$ expansion and also called the planar QCD flavor symmetry [8], it restricts the complete Lagrangian of the theory and give us the $1/N_c$ baryon chiral Lagrangian at lowest order as [30]

$$\mathcal{L}_{baryon} = \mathbf{B}^\dagger \left[i\mathcal{D}^0 - M_{hyperfine} + \mathrm{Tr}(\mathcal{A}^k \lambda^c) A^{kc} + \frac{1}{N_c} \mathrm{Tr}\left(\mathcal{A}^k \frac{2I}{\sqrt{6}} \right) A^k \right] \mathbf{B}, \tag{4.1}$$

where the derivative is

$$\mathcal{D}^0 = \partial \mathbb{1} + \mathrm{Tr}(\mathcal{V}^0 \lambda^c) T^c, \tag{4.2}$$

$\mathbf{B}$ and $\mathbf{B}^\dagger$ are the baryon fields and the terms inside the parenthesis are the lowest order contributions to the lagrangian plus higher-order contribution that have been ignored in this work. All the terms that involve baryons in the Lagrangian, can be expanded using the spin flavor generators (3.23), the flavor indices run from 1 to 9 because we are including the full meson nonet (3.2) plus the η'. The operator $\mathcal{M}$ denotes the spin splittings of baryon states.

Also, the axial vector and vector combinations of the meson fields (3.3) that appear in the Lagrangian have components

$$\mathcal{V}^0 = \frac{1}{2}\left(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi\right), \qquad \mathcal{A}^k = \frac{i}{2}\left(\xi\partial^k\xi^\dagger - \xi^\dagger\partial^k\xi\right). \tag{4.3}$$

As we said, all the QCD operators involved in the Lagrangian have a well-defined $1/N_c$ expansion, particularly the baryon axial vector current A^{kc} is a spin-1 object, odd under time reversal and an octet under $SU(3)$ that couples the pion axial combination with the baryons, A^{kc} is given by

$$A^{kc} = \left\langle B' \left| \left(\bar{q}\gamma^k\gamma_5\frac{\lambda^a}{2}q\right)_{QCD} \right| B \right\rangle, \tag{4.4}$$

which has the same expansion than the one given in (3.30), and in particular for the physical value $N_c = 3$ it only has four terms as

$$A^{ia} = a_1 G^{ia} + b_2\frac{1}{N_c}\mathcal{D}_2^{ia} + b_3\frac{1}{N_c^2}\mathcal{D}_3^{ia} + c_3\frac{1}{N_c^2}\mathcal{O}_3^{ia}. \tag{4.5}$$

For the octet of baryons, the axial vector coupling is $g_A \approx 1.27$. On the other hand, the operator A^k is a spin-1 object, a singlet under $SU(3)$ defined by

$$A^{kc} = \left\langle B' \left| \left(\bar{q}\gamma^k\gamma_5\frac{I}{2}q\right)_{QCD} \right| B \right\rangle, \tag{4.6}$$

and has a $1/N_c$ expansion as

$$A^k = \sum_{n=1,3}^{N_c} b_n^{1,1}\frac{1}{N_c^{n-1}}\mathcal{D}_n^k \tag{4.7}$$

with $\mathcal{D}_1^k = J^k$ and its recursion formula $D_{2m+1}^k = \{J^2, J^k\}$ for $m \geq 1$, in the particular case of $N_c = 3$ the expansion is truncated to

$$A^k = b_1^{1,1}J^k + b_3^{1,1}\frac{1}{N_c^2}\{J^2, J^k\}. \tag{4.8}$$

To describe the operator $\mathcal{M}_{hyperfine}$ we will use the expansion of the mass operator given by [8]

$$\mathcal{M} = m_0 N_c \mathbb{1} + \sum_{n=2,4}^{N_c-1} m_n\frac{1}{N^{n-1}}J^n, \tag{4.9}$$

the m_n coefficients are unknown, but considering only the physical case $N_c = 3$ and as the first term in the expansion is the spin-independent mass of the baryon multiplet, we can remove it by the heavy baryon field redefinition, then the hyperfine mass operator is only

$$\mathcal{M}_{hyperfine} = \frac{m_2}{N_c}J^2. \tag{4.10}$$

As important remarks, the N_c counting rules for the theory, in the case of baryons with spin of order unity are summarized in

$$T^a \sim N_c, \qquad G^{ia} \sim N_c, \qquad J^i \sim 1, \tag{4.11}$$

also, the pion decay constant $f \sim N_c$. Further, the n-body operators satisfy some properties: In general, the product of a n-body operator and a m-body operator is a $(n+m)$-body operator, but the commutator structure implies

$$[\mathcal{O}^{(n)}, \mathcal{O}^{(m)}] = \mathcal{O}^{(n+m-1)}, \tag{4.12}$$

so, the commutator reduces the number of bodies of the operator by 1 [31][20][28].

4.2 Renormalization of the axial current in the HBChPT and the large N_c limit

One of the first applications of the $1/Nc$ baryon chiral Lagrangian is the renormalization of the Baryon axial vector current using non-analytic meson-loop corrections [8], here we only consider the one loop corrections to the matrix element $\langle B'|A^{kc}|B\rangle$ by the diagrams presented in the past chapter. First, the renormalization of the wave function at one-loop is given by the diagram in fig. 3.1 that implies [20]

$$z_{B'B} = -\sum_{j,b,B_I} [A^{jb}]_{B'B_I}[A^{jb}]_{B_IB}\frac{\partial F(m_b, \Delta_{B_IB}, \mu)}{\partial \Delta_{B_IB}} \tag{4.13}$$

As in our analysis of the renormalization of the axial current in HBChPT the contribution of the loops will be given in therms of the integral $F(m_\Pi, \Delta, \mu)$. Working again with fig. 3.2, the diagram (a) contributes with a vertex inside the loop as [20]

$$\begin{aligned}[\delta A^{kc}]^{vertex}_{B'B} = &-\sum_{j,b,B_1,B_2} \frac{i}{f^2}[A^{ib}]_{B'B_2}[A^{kc}]_{B_2B_1}[A^{jb}]_{B_1B} \\ &\times \int \frac{d^4k}{(2\pi)^4}\frac{(k^i)(-k^j)}{(k^2-m_b^2)[k\cdot v-(M_1-M)+i\epsilon][(k-q)\cdot v-(M_2-M)+i\epsilon]},\end{aligned} \tag{4.14}$$

where q is the momentum of the axial vector vertex. The value of the product $q \cdot v$ depends on the type of transition of the baryons in the matrix elements, in the case of octet-octet transition $q \cdot v = 0$ and for the decuplet-decuplet $q \cdot v = M - M'$ that is the average decuplet-octet mass difference. To solve the last integral a useful relation is

$$\frac{1}{(k^0-\Delta_1+i\epsilon)(k^0-\Delta+i\epsilon)} = \frac{1}{\Delta_1-\Delta_2}\left[\frac{1}{k^0-\Delta+i\epsilon}-\frac{1}{k^0-\Delta+i\epsilon}\right] \tag{4.15}$$

using this relation is possible to recover a structure of the integral $F(m, \Delta, \mu)$ and the correction for this diagram reduces to

$$\begin{aligned}[\delta A^{kc}]^{vertex}_{B'B} = &-\sum_{j,b,B_1,B_2} \frac{i}{f^2}[A^{ib}]_{B'B_2}[A^{kc}]_{B_2B_1}[A^{jb}]_{B_1B} \\ &\times \frac{F(m_b, \Delta_{B_1B}, \mu) - F(m_b, \Delta_{B_2B'}, \mu)}{\Delta_{B_1B}-\Delta_{B_2B'}}.\end{aligned} \tag{4.16}$$

Working on the diagrams (b) and (c), it is easy to see that both cases have a separate contribution of the renormalizacion of the wave function and from the axial current vertex, so the total contribution from these diagrams is

$$-\frac{1}{2}\left(\sum_{B_1} z_{B'B_1}[A^k c]_{B_1 B} + \sum_{B_2}[A^{kc}]_{B'B_2} z_{B_2 B}\right)$$

Then, the correction as contribution from (a, b, c) in fig 3.2 is [30]

$$\begin{aligned}
[\delta A^{kc}]_{B'B} &= [\delta A^{kc}]_{B'B} - \frac{1}{2}\left(\sum_{B_1} z_{B'B_1}[A^k c]_{B_1 B} + \sum_{B_2}[A^{kc}]_{B'B_2} z_{B_2 B}\right) \\
&= -\sum_{j,b,B_1,B_2}[A^{ib}]_{B'B_2}[A^{kc}]_{B_2 B_1}[A^{jb}]_{B_1 B}\frac{F(m_b,\Delta_{B_1 B},\mu) - F(m_b,\Delta_{B_2 B'},\mu)}{\Delta_{B_1 B} - \Delta_{B_2 B'}} \\
&+ \frac{1}{2}\sum_{j,b,B_1,B_2}[A^{jb}]_{B'B_2}[A^{ib}]_{B_2 B_1}[A^{kc}]_{B_1 B}\frac{\partial F(m_b,\Delta_{B_2 B_1},\mu)}{\partial \Delta_{B_2 B_1}} \\
&+ \frac{1}{2}\sum_{j,b,B_1,B_2}[A^{kc}]_{B'B_2}[A^{jb}]_{B_2 B_1}[A^{jb}]_{B_1 B}\frac{\partial F(m_b,\Delta_{B_1 B},\mu)}{\partial \Delta_{B_1 B}}.
\end{aligned} \tag{4.17}$$

Taking the degeneracy limit of the octet and decuplet baryons the mass differences Δ_{AB} goes to zero, it implies that

$$\frac{F(m_b,\Delta_{B_1 B},\mu) - F(m_b,\Delta_{B_2 B'},\mu)}{\Delta_{B_1 B} - \Delta_{B_2 B'}} \to F^{(1)}(m_b,0,\mu),$$

and applying the more compact notation $[A^{kc}]_{B'B} \to A^{kc}$ where the summation over the baryon state is an implicit matrix multiplication and using the Einstein convention to sum over the repeated indices, so the complete correction is

$$\delta A^{kc} = F^{(1)}(m_b,0,\mu)\left(-A^{jb}A^{kc}A^{jb} + \frac{1}{2}A^{jb}A^{jb}A^{kc} + \frac{1}{2}A^{kc}A^{jb}A^{jb}\right) \tag{4.18}$$

putting the result in a more suitable form by doing some algebra of commutators

$$\frac{1}{2}F^{(1)}(m_b,0,\mu)[A^{jb},[A^{jb},A^{kc}]]. \tag{4.19}$$

Now, the correction that contains a full dependence on Δ/m is [31]

$$\begin{aligned}
\delta A_1^{kc} &= \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab}_{(1)} - \frac{1}{2}\left\{A^{ja},\left[A^{kc},\left[\mathcal{M},A^{jb}\right]\right]\right\}\Pi^{ab}_{(2)} \\
&+ \frac{1}{6}\left(\left[A^{ja},\left[\left[\mathcal{M},\left[\mathcal{M},A^{jb}\right]\right],A^{kc}\right]\right] - \frac{1}{2}\left[\left[\mathcal{M},A^{ja}\right],\left[\left[\mathcal{M},A^{jb}\right],A^{kc}\right]\right]\right)\Pi^{ab}_{(3)} + \ldots
\end{aligned} \tag{4.20}$$

It could be probed using a Taylor expansion, also, the tensor $\Pi^{ab}_{(n)}$ is symmetric and is composed of the meson loop integrals where we supposed that is emitted a meson with flavor a and then a meson of flavor b is reabsorbed. Using the linearly independent combinations of the integral $F(m_\Pi,0,\mu)$, $\Pi^{ab}_{(n)}$ is decomposed as

$$\Pi^{ab}_{(n)} = F^{(n)}_{\mathbf{1}}\delta^{ab} + F^{(n)}_{\mathbf{8}}d^{ab8} + F^{(n)}_{\mathbf{27}}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right], \tag{4.21}$$

where

$$F_{\mathbf{1}}^{(n)} = \frac{1}{8}\left[3F^{(n)}(m_\pi,0,\mu)+4F^{(n)}(m_K,0,\mu)+F^{(n)}(m_\eta,0,\mu)\right] \tag{4.22}$$

$$F_{\mathbf{8}}^{(n)} = \frac{2\sqrt{3}}{5}\left[\frac{3}{2}F^{(n)}(m_\pi,0,\mu)-F^{(n)}(m_K,0,\mu)-\frac{1}{2}F^{(n)}(m_\eta,0,\mu)\right] \tag{4.23}$$

$$F_{\mathbf{27}}^{(n)} = \frac{1}{3}F^{(n)}(m_\pi,0,\mu)-\frac{4}{3}F^{(n)}(m_K,0,\mu)+F^{(n)}(m_\eta,0,\mu) \tag{4.24}$$

we have taken the limit Δ/m_Π of the function $F^{(n)}(m_\Pi,\Delta,\mu)$, defined by

$$F^{(n)}(m_\Pi,\Delta,\mu) \equiv \frac{\partial^n F(m_\Pi,\Delta,\mu)}{\partial\Delta^n}, \tag{4.25}$$

these functions are well-defined for the first values of n. In the degeneracy limit $\Delta/m_\Pi = 0$ the integrals $F^{(n)}(m_\Pi,0,\mu)$ are reduced to

$$F^{(1)}(m_\Pi,0,\mu) = \frac{m_\Pi^2}{16\pi^2 f^2}\left(\lambda_\epsilon + 1 - \ln\frac{m_\Pi^2}{\mu^2}\right), \tag{4.26}$$

$$F^{(2)}(m_\Pi,0,\mu) = -\frac{m_\Pi}{8\pi f^2}. \tag{4.27}$$

where

$$\lambda_\epsilon = \frac{2}{\epsilon} - \gamma + \ln(4\pi), \tag{4.28}$$

with $\gamma \simeq 0.577216$ the Euler constant. The corrections of the axial vector current has been constructed considering the (a,b,c) diagrams so far, and is possible to see that the structure of the corrections became more complicated as the order of the symmetric tensor $\Pi^{ab}_{(n)}$ became higher, the complication of the structure appears as addition of more commutators and anti-commutators per term. As an example, the first terms of the correction for the axial current, at first order in the expansion 4.20 given by the contribution of flavor singlet, octet and **27** are [12]:

- Flavor singlet contribution

$$\begin{aligned}
[A^{ia},[A^{ia},A^{kc}]] = &\left[\frac{23}{12}a_1^3 - \frac{2(N_c+3)}{3N_c}a_1^2b_2 + \frac{N_c^2+6N_c-54}{6N_c^2}a_1b_2^2\right.\\
&\left. - \frac{N_c^2+N_c+2}{N_c^2}a_1^2b_3 - \frac{N_c^2+6N_c-3}{N_c^2}a_1^2c_3 - \frac{12(N_c+3)}{N_c^3}a_1b_2b_3\right]G^{kc}\\
&+ \frac{1}{N_c}\left[\frac{101}{12}a_1^2b_2 + \frac{4(N_c+3)}{3N_c}a_1b_2^2 - \frac{3(N_c+3)}{N_c}a_1^2b_3\right.\\
&\left. - \frac{N_c+3}{2N_c}a_1^2c_3 + \frac{N_c^2+6N_c+2}{N_c^2}a_1b_2b_3 - \frac{3(N_c^2+6N_c-24)}{2N_c^2}a_1b_2c_3\right]\mathcal{D}_2^{kc}\\
&+ \frac{1}{N_c^2}\left[\frac{11}{4}a_1b_2^2 + \frac{51}{4}a_1^2b_3 + 2a_1^2c^3\frac{17(N_c+3)}{3N_c}a_1b_2b_3 - \frac{9(N_c+3)}{2N_c}a_1b_2c_3\right]\mathcal{D}_3^{kc} + \mathcal{O}(4)
\end{aligned} \tag{4.29}$$

- Flavor octet contribution

$$
\begin{aligned}
&d^{ab8}[A^{ia},[A^{ib},A^{kc}]] = \\
&\left[\frac{11}{24}a_1^3 - \frac{2(N_c+3)}{3N_c}a_1^2b_2 - \frac{9}{2N_c^2}a_1b_2^2 - \frac{5}{N_c^2}a_1^2b_3 + \frac{3}{2N_c^2}a_1^2c_3 - \frac{6(N_c+3)}{N_c^3}a_1b_2b_3\right]d^{c8e}G^{ke} \\
&+ \frac{1}{8N_c}\left[-\frac{2(N_c+3)}{N_c}(6a_1^2b_3+a_1^2c_3) - \frac{12}{N_c^2}(b_2^3+2a_1b_2b_3-12a_1b_2c_3) + 23a_1^2b_2\right]d^{ce8}\mathcal{D}_2^{kc} \\
&- \frac{1}{6N_c}\left[4a_1^2b_2 + \frac{N_c+3}{N_c}(a_1b_2^2+6a_1^2b_3+6a_1^2b_3+6a_1^2c_3+\frac{36}{N_c^2}a_1b_2b_3)\right]\{G^{kc,T^8}\} \\
&+ \frac{1}{6N_c}\left[11a_1^2b_2 + \frac{2(N_c+3)}{N_c}a_1b_2 + \frac{48}{N_c^2}a_1b_2c_3\right]d^{c8e}\{G^{k8},T^c\} \\
&+ \frac{1}{24N_c^2}\left[27a_1b_2^2 + 65a_1^2b_3 + 8a_1^2c_3 + \frac{36(N_c+3)}{N_c}a_1b_2b_3 - \frac{46(N_c+3)}{N_c}a_1b_2c_3\right]d^{c8e}\mathcal{D}_3^{ke} \qquad (4.30) \\
&+ \frac{1}{6N_c^2}\left[3a_1b_2^2 - 2a_1^2b_3 + 30a_1^2c_3 + \frac{4(N_c+3)}{N_c}a_1b_2b_3\right]\{G^{kc},\{J^r,G^{r8}\}\} \\
&+ \frac{1}{6N_c^2}\left[3a_1b_2^2 + 28a_1^2b_3 - 15a_1^2c_3 + \frac{4(N_c+3)}{N_c}a_1b_2b_3\right]\{G^{k8},\{J^r,G^{rc}\}\} \\
&+ \frac{1}{3N_c^2}\left[12a_1^2b_3 - 2a_1^2c_3 - \frac{2(N_c+3)}{N_c}a_1b_2c_3\right]\{J^k,\{G^{rc},G^{r8}\}\} \\
&+ \frac{1}{12N_c^2}\left[2a_1b_2^2 - 9a_1^2b_3 - \frac{3}{2}a_1^2c_3 - \frac{N_c+3}{N_c}(b_2^3 -{}^a_1\, b_2b_3 + 9a_1b_2c_3)\right]\{J^k,\{T^c,T^8\} + \mathcal{O}(4)
\end{aligned}
$$

- Flavor **27** contribution

$$
\begin{aligned}
&[A^{i8},[A^{i8},A^{kc}]] = \\
&\left[\left(\frac{1}{4}a_1^3 - \frac{1}{N_c^2}(2a_1b_2^2+2a_1^2b_3-a_1^2c_3)\right)f^{c8e}f^{8eg} + \frac{1}{2}\left(a_1^3 + \frac{1}{N_c^2}(2a_1^2b_3-a_1^2c_3)\right)d^{c8e}d^{8eg}\right]G^{kg} \\
&+ \frac{1}{N_c}\left[\frac{1}{12}a_1^2b_2(4\delta^{cg}+21f^{c8e}f^{8eg}) + \frac{1}{N_c^2}(-b_2^3+9a_1b_2c_3)f^{c8e}f^{8eg}\right]\mathcal{D}_2^{kg} \\
&+ \frac{1}{2N_c}a_1^2b_2(2d^{c8e}\{G^{ke},T^8\} + d^{88e}\{G^{ke},T^c\}) + \frac{1}{Nc}\left(a_1^2b_2 + \frac{4}{N_c^2}a_1b_2b_3if^{c8e}[G^{k8},\{J^r,G^{re}\}]\right) \\
&+ \frac{1}{12N_c^2}[9a_1b_2^2f^{c8e}f^{8eg} + a_1^2B_3(8\delta^{cg}+9f^{c8g}f^{8eg}+6d^{c8e}d^{8eg}) + 6a_1^2c_3d^{c8e}d^{8eg}]\mathcal{D}^kg_3 \\
&+ \frac{1}{2N_c^2}a_1b_2^2(\{G^{kc},\{T^8,T^8\}\} + 2\{G^{k8},\{T^c,T^8\}\}) + \frac{1}{N_c^2}(4a_1^2b_3-a_1^2c_3)d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\} \\
&- \frac{1}{2N_c^2}(6a_1^2b_3+a_1^2c_3)d^{c8e}\{J^k,\{G^{re},G^{r8}\}\} + \frac{1}{N_c^2}(2a_1^2b_3-a_1^2c_3)\{G^{rc}.\{G^{r8},G^{k8}\}\} \\
&- \frac{1}{N_c^2}(2a_1^2b_3+a_1^2c_3)\{G^{kc},\{G^{r8},G^{r8}\}\} + \frac{1}{2N_c^2}(-2a_1^2b_3+3a_1^{@}c_3)d^{c8e}\{G^k8,\{J^r,G^{re}\}\} \\
&+ \frac{1}{2N_c^2}(2a_1^2b_3-a_1^2c_3)(d^{88e}\{J^k,\{G^{rc},G^{re}\}\} + d^{88e}\{G^{ke},\{J^r,G^{rc}\}\}) + \mathcal{O}(4),
\end{aligned}
$$

(4.31)

where $\mathcal{O}(4)$ indicates the 4-body or more operators. To obtain the above results, it was necessary to apply each commutator in a quark basis, then expand it in a well-known complete operator basis previously constructed in [12]. Now, to include the contribution of the last diagram, its correction is given by [12]

$$\delta A_2^{kc} = -\frac{1}{2}\left[T^a, [T^b, A^{kc}]\right]\Pi^{ab}, \tag{4.32}$$

here the tensor Π^{ab} is also symmetric and posses an expansion similar to $\Pi^{ab}_{(n)}$

$$\Pi^{ab} = I_{\mathbf{1}}\delta^{ab} + I_{\mathbf{8}}d^{ab8} + I_{\mathbf{27}}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right], \tag{4.33}$$

again, the flavor singlet $I_{\mathbf{1}}$, flavor octet $I_{\mathbf{8}}$ and flavor **27** $I_{\mathbf{27}}$ are linear combinations of a loop integral, in this case

$$I(m_\Pi, \mu) = \frac{i}{f^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_\Pi} = \frac{m_\Pi^2}{16\pi^2 f^2}\left[\ln\frac{m_\Pi^2}{\mu^2} - 1\right], \tag{4.34}$$

and its respective linear combinations

$$I_{\mathbf{1}} = \frac{1}{8}\left[3I(m_\pi, \mu) + 4I(m_K, \mu) + I(m_\eta, \mu)\right], \tag{4.35}$$

$$I_{\mathbf{8}} = \frac{2\sqrt{3}}{5}\left[\frac{3}{2}I(m_\pi, \mu) + I(m_K, \mu) - \frac{1}{2}I(m_\eta, \mu)\right], \tag{4.36}$$

$$I_{\mathbf{27}} = \frac{1}{3}I(m_\pi, \mu) - \frac{4}{3}I(m_K, \mu) + I(m_\eta, \mu), \tag{4.37}$$

also it is possible to compute the correction from the flavor singlet, octet, and **27** contribution [12]

- Flavor singlet contribution
$$[T^a, [T^a, A^{kc}]] = 3A^{kc} \tag{4.38}$$

- Flavor octet contribution
$$d^{ab8}[T^a, [T^b, A^{kc}]] = \frac{3}{2}d^{c8e}A^{kc} \tag{4.39}$$

- Flavor **27** contribution
$$[T^8, [T^8, A^{kc}]] = f^{c8e}f^{8eg}A^{kg} \tag{4.40}$$

and the total one loop in the limit $\Delta/m_\Pi = 0$ is

$$\delta A^{kc} = \frac{1}{2}[A^{ja}, [A^{jb}, A^{kc}]]\Pi^{ab}_{(1)} - \frac{1}{2}[T^a, [T^b, A^{kc}]]\Pi^{ab}, \tag{4.41}$$

this is the complete contribution as 1-loop corrections to the baryon axial vector current at leading order. It is possible to include more corrections, for example the symmetry broken corrections or contribution from other representations, but in this work only the 1-loop corrections will be considered [32][31][28][12].

Chapter 5

Results

The complete one-loop correction to the baryon axial vector current have never been explicitly computed, it was calculated only at certain order but not considering the complete expression of the axial current in terms of n-body operators.

In this chapter, the one-loop contributions from the singlet and octet representation to the axial current will be presented and this result will be compared with the result obtained by chiral perturbation theory to show the power of calculation of the heavy baryon chiral perturbation theory combined with the large-N_c formalism. The **27** flavor representation has not been considered in this work, it only has been partially computed and included in Appendix B.

All the results that will be presented in this chapter have been computed taking $\Delta = 0$.

5.1 Complete loop corrections to the axial current

To compute the complete one-loop correction of the baryon axial vector current, it is necessary to consider the complete expansion in n-body terms given by (4.5), only considering the physical value of $N_c = 3$, and replacing it in the loop correction (4.20). The first order contribution to (4.20) is given by the commutator:

$$\left[A^{ja}, \left[A^{jb}, A^{kc}\right]\right], \tag{5.1}$$

so, all the contributions will be given in terms of commutators as (5.1) involving operators G^{kc}, $\mathcal{D}_2^{kc}$, $\mathcal{D}_3^{kc}$ and $\mathcal{O}_3^{kc}$ that also are divided in flavor singlet, octet and **27** representation contributions, the complete expansion is in Appendix B.

With all the commutators expanded in a basis of spin-flavor operators it is possible to compute the matrix elements in different processes, we only need to include the coefficients a_1, b_2, b_3 and c_3 to have the complete expansion of the axial current in the large N_c limit. Some examples are presented below.

The singlet contribution of the matrix element of the renormalized axial current of the neutron-proton process is given by

$$\begin{aligned} \langle n|A^{kc}|p\rangle_{\mathbf{1}} = {} & \frac{115}{144}a_1^3 + \frac{7}{48}a_1^2 b_2 + \frac{19}{48}a_1 b_2^2 + \frac{7}{144}b_2^3 - \frac{169}{216}a_1 b_2 b_3 + \frac{19}{144}b_2^2 b_3 \\ & + \frac{247}{432}a_1 b_3^2 + \frac{169}{1296}b_2 b_3^2 + \frac{247}{3888}b_3^3 - \frac{11}{12}a_1^2 c_3 - \frac{37}{36}a_1 b_2 b_3 - \frac{193}{108}a_1 b_3 c_3 \\ & - \frac{11}{48}a_1 c_3^2 - \frac{37}{144}b_2 c_3^2 - \frac{193}{432}b_3 c_3^2, \end{aligned} \tag{5.2}$$

and the octet contribution is

$$\begin{aligned}\langle n|A^{kc}|p\rangle_{\mathbf{8}} &= \frac{55}{288\sqrt{3}}a_1^3 - \frac{127}{288}a_1^2b_2 - \frac{11}{288}a_1b_2^2 - \frac{5}{288}b_2^3 - \frac{419}{864\sqrt{3}}a_1^2b_3 - \frac{109}{432}a_1b_2b_3 \\ &- \frac{11}{864\sqrt{3}}b_2^2b_3 + \frac{3\sqrt{3}}{32}a_1b_3^2 + \frac{109}{2592\sqrt{3}}b_2b_3^2 + \frac{1}{32\sqrt{3}}b_3^3 - \frac{47}{72\sqrt{3}}a_1^2c_3 \\ &- \frac{59}{72\sqrt{3}}a_1b_2b_3 - \frac{287}{216\sqrt{3}}a_1b_3c_3 - \frac{47}{288\sqrt{3}}a_1c_3^2 - \frac{59}{288\sqrt{3}}b_2c_3^2 - \frac{287}{864\sqrt{3}}b_3c_3^2.\end{aligned} \tag{5.3}$$

Other example is the $\Lambda - \Sigma^{+/-}$ process, where the singlet contribution of the renormalized axial current is given by

$$\begin{aligned}\langle\Lambda|A^{kc}|\Sigma^{+/-}\rangle_{\mathbf{1}} &= \frac{23}{24\sqrt{6}}a_1^3 - \frac{\sqrt{2}}{3\sqrt{3}}a_1^2b_2 + \frac{5}{24\sqrt{6}}a_1b_2^2 + \frac{5}{9\sqrt{6}}a_1b_2b_3 + \frac{5}{72\sqrt{6}}b_2^2b_3 \\ &+ \frac{47}{72\sqrt{6}}a_1b_3^2 + \frac{5}{54\sqrt{6}}b_2b_3^2 + \frac{47}{648\sqrt{6}}b_3^3 - \frac{1}{\sqrt{6}}a_1^2c_3 - \frac{\sqrt{3}}{2\sqrt{2}}a_1b_2b_3 \\ &- \frac{13}{9\sqrt{6}}a_1b_3c_3 - \frac{1}{4\sqrt{6}}a_1c_3^2 - \frac{\sqrt{3}}{8\sqrt{2}}b_2c_3^2 - \frac{13}{36\sqrt{6}}b_3c_3^2,\end{aligned} \tag{5.4}$$

while the octet contribution is

$$\begin{aligned}\langle\Lambda|A^{kc}|\Sigma^{+/-}\rangle_{\mathbf{8}} &= \frac{11}{144\sqrt{2}}a_1^3 - \frac{\sqrt{2}}{9}a_1^2b_2 - \frac{1}{48\sqrt{2}}a_1b_2^2 - \frac{71}{432\sqrt{2}}a_1^2b_3 - \frac{1}{18\sqrt{2}}a_1b_2b_3 \\ &- \frac{1}{144\sqrt{2}}b_2^2b_3 - \frac{1}{432\sqrt{2}}a_1b_3^2 + \frac{1}{108\sqrt{2}}b_2b_3^2 - \frac{1}{3888\sqrt{2}}b_3^3 + \frac{1}{12\sqrt{2}}a_1^2c_3 \\ &- \frac{5}{36\sqrt{2}}a_1b_2b_3 - \frac{17}{108\sqrt{2}}a_1b_3c_3 + \frac{1}{48\sqrt{2}}a_1c_3^2 - \frac{5}{144}b_2c_3^2 - \frac{17}{432\sqrt{2}}b_3c_3^2,\end{aligned} \tag{5.5}$$

these results were obtained using the large N_c expansion, with the physical values of $N_c = 3$ and $N_f = 3$.

5.2 Comparison between ChPT and HBChPT in the large-N_c limit

Now, as the chiral coefficients are related with the large N_c coefficients at $N_c = 3$ for the axial current by [8]

$$\begin{aligned}D &= \frac{1}{2}a_1 + \frac{1}{6}b_3, \\ F &= \frac{1}{3}a_1 + \frac{1}{6}b_2 + \frac{1}{9}b_3, \\ \mathcal{C} &= -a_1 - \frac{1}{2}c_3, \\ \mathcal{H} &= -\frac{3}{2}a_1 - \frac{3}{2}b_2 - \frac{5}{2}b_3.\end{aligned} \tag{5.6}$$

Using these coefficients is possible to compute the matrix elements by the constants defined in chiral perturbation theory for each process. Some processes are presented below: In the case of the neutron-proton matrix element, the flavor contributions are given as linear combinations of the following factors [31]

$$a_{pn}^{(1)} = \frac{1}{4}\left(F+D\right)^3 + \frac{16}{9}\left(F+D\right)\mathcal{C}^2 - \frac{50}{81}\mathcal{H}\mathcal{C}^2 - \left(\frac{9}{4}\left(F+D\right)^2 + 2\mathcal{C}^2\right)\left(D+F\right), \tag{5.7}$$

$$\begin{aligned} a_{pn}^{(2)} =& \frac{1}{3}\left(-3F^3 + 3F^2D - FD^2 + D^3\right) + \frac{1}{9}\left(F+3D\right)\mathcal{C}^2 - \frac{10}{81}\mathcal{H}\mathcal{C}^2 \\ & - \left(\frac{1}{2}\left(9F^2 - 6FD + 5D^2\right)\right)\left(D+F\right), \end{aligned} \tag{5.8}$$

$$a_{pn}^{(3)} = -\frac{1}{12}\left(F+D\right)\left(3F-D\right)^2 - \frac{1}{4}\left(3F-D\right)^2\left(D+F\right). \tag{5.9}$$

The complete flavor singlet contribution is expressed as a linear combination of the coefficients as

$$\langle n|A^{kc}|p\rangle_{\mathbf{1}} = -\left(a_{pn}^{(1)} + a_{pn}^{(2)} + a_{pn}^{(3)}\right), \tag{5.10}$$

and the flavor octet contribution is given by

$$\langle n|A^{kc}|p\rangle_{\mathbf{8}} = -\frac{1}{\sqrt{3}}\left[a_{pn}^{(1)} - \frac{1}{2}a_{pn}^{(2)} - a_{pn}^{(3)}\right]. \tag{5.11}$$

In the case of the $\Lambda - \Sigma^{+/-}$ matrix element, the flavor contributions are given as linear combinations of the following factors [31]

$$\begin{aligned} a_{\Lambda\Sigma^{+/-}}^{(1)} =& \frac{2}{3\sqrt{6}}D\left(6F^2 - D^2\right) + \frac{2}{3\sqrt{6}}\mathcal{C}^2\left(2F + \frac{1}{3}D\right) - \frac{10}{27\sqrt{6}}\mathcal{H}\mathcal{C}^2 \\ & - \frac{2}{\sqrt{6}}D\left(3F^2 + 2D^2 + \frac{11}{12}\mathcal{C}^2\right), \end{aligned} \tag{5.12}$$

$$\begin{aligned} a_{\Lambda\Sigma^{+/-}}^{(2)} =& -\frac{1}{\sqrt{6}}D(F^2 - D^2) + \frac{8}{3\sqrt{6}}\mathcal{C}^2\left(F + \frac{2}{3}D\right) - \frac{5}{27\sqrt{6}}\mathcal{H}\mathcal{C}^2 \\ & \frac{2}{\sqrt{6}}D\left(6F^2 + 2D^2 + \frac{4}{3}\mathcal{C}^2\right) \end{aligned} \tag{5.13}$$

$$a_{\Lambda\Sigma^{+/-}}^{(3)} = \frac{2}{3\sqrt{6}}D(D^2 + \mathcal{C}^2) - \frac{2}{\sqrt{6}D}\left(D^2 + \frac{1}{4}\mathcal{C}^2\right). \tag{5.14}$$

The complete flavor singlet contribution is expressed as a linear combination of the coefficients as

$$\langle\Lambda|A^{kc}|\Sigma^{+/-}\rangle_{\mathbf{1}} = -\left(a_{\Lambda\Sigma^{+/-}}^{(1)} + a_{\Lambda\Sigma^{+/-}}^{(2)} + a_{\Lambda\Sigma^{+/-}}^{(3)}\right), \tag{5.15}$$

and the flavor octet contribution is given by

$$\langle\Lambda|A^{kc}|\Sigma^{+/-}\rangle_{\mathbf{8}} = -\frac{1}{\sqrt{3}}\left[a_{\Lambda\Sigma^{+/-}}^{(1)} - \frac{1}{2}a_{\Lambda\Sigma^{+/-}}^{(2)} - a_{\Lambda\Sigma^{+/-}}^{(3)}\right]. \tag{5.16}$$

These results are totally equivalent to the results given from Eq. (5.2) to Eq. (5.5), for the values of the chiral coefficients given in (5.6). In other words, both approches are equivalent for $N_c = 3$.

Conclusions

In this work has been computed the complete 1-loop renormalization for the singlet and octect flavor contributions at leading order of the baryon axial vector current in the combined formalism of heavy baryon chiral perturbation theory in the large N_c limit, based in an expansion on the spin-flavor symmetry and its operators for the physical value $N_c = 3$ and taking $\Delta = 0$ which is the baryon mass splitting. Then, these results were compared with the heavy baryon chiral perturbation theory through the computation of the matrix elements for $SU(6)$ states.

The contribution given by the **27** flavor representation has been partially computed, but it is still unfinished because it represents a more complicated computational problem than the other representations. The operator reductions for this contribution has been included in the Appendix B, but as it is incomplete then it was impossible to include its comparison with HBChPT.

From the results, it is possible to conclude that the renormalization of the axial current in the heavy baryon chiral perturbation theory in the large N_c limit is totally equivalent to the result for the renormalization procedure only using the heavy baryon chiral perturbation theory formalism even though the calculation procedures are too different, here has been considered for first time the complete replacement of the expansion in spin flavor operators at $N_c = 3$ in the 1-loop correction at leading order and that is the reason which makes possible to found the total equivalence between the two formalisms. The importance of found the equivalence between that the chiral formalism with and without considering the large N_c limit lies in the fact that the large N_c framework give us a very systematic expansion of every QCD operator and this systematic expansion does not appear in the heavy baryon chiral perturbation theory, that is a more intuitive and technical formalism, so is more difficult to work in there.

As future perspectives for this work are considered the complete computation of the renormalization including the **27** contribution, then the numerical evaluation of the chiral coefficients D, F, $\mathcal{C}$ and $\mathcal{H}$. Moreover, the results from this work will be useful for the computation of the quadrupole momentum operator in the large-N_c limit

Appendix A

Lie Algebras and Group Theory

The mathematical structure of the gauge theories is based in the Lie algebras and Lie groups, that is the reason why this is a necessary topic for all of the theoretical physicists in high energy physics and related areas. In particular, this work is based on the $SU(N)$ group, that is the special unitary Lie group, this is one of the must important groups in particle physics and interesting quantities can be expressed as a representation of this group as the spin, flavor, color, etc. Some important definitions and structures are presented bellow [33].

Definition 1. An algebra $\mathfrak{A}$ is a vector space endowed with an additional binary operation $\bullet : \mathfrak{A} \times \mathfrak{A} \to \mathfrak{A}$ that only needs to be bilinear. i.e. $\bullet$ satisfies:

$$(x + y) \bullet z = x \bullet y + x \bullet z, \tag{A.1}$$

$$x \bullet (y + z) = x \bullet y + x \bullet z \tag{A.2}$$

Definition 2. A *Lie algebra* $\mathfrak{g}$ is an algebra in where the bilinear operation is the *Lie bracket* $[\cdot, \cdot] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$, it has two important properties, *antisymmetry* and *Jacobi identity* denoted as:

$$[x, y] = -[y, x], \tag{A.3}$$

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0, \tag{A.4}$$

for all x, y and $z \in \mathfrak{g}$.

If the Lie algebra is an associative algebra with a product $*$, the lie bracket is the *commutator* with respect to the original product, it is:

$$[x, y] := x * y - y * x \tag{A.5}$$

As a Lie algebra $\mathfrak{g}$ is a vector space, if it has dimension d then it will be expanded in terms of a basis B such that

$$B = \{T^a | a = 1, 2, ...d\} \tag{A.6}$$

where T^a are the generators of the algebra and every element in $\mathfrak{g}$ can be expanded in terms of them. Through the Lie bracket, the generator of the algebra follow:

$$[T^a, T^b] = f^{abc} T^c, \tag{A.7}$$

the f^{abc} tensors are called the *structure constants* of the algebra $\mathfrak{g}$, these constants are elements of the *adjoint representation* of the algebra [34].

Remark. An *abelian* Lie algebra is a Lie algebra which satisfies $[\mathfrak{g}, \mathfrak{g}] = 0$.

The other important structure in this works are the Lie groups, but before define it is important to introduce some previous concepts.

Definition 3. A *group* g is a set of elements $\{g_i\}$ also called group operations, with a multiplication operation $\circ$ that satisfy the next axioms:

- Clousure. $g_i \circ g_j \in g$ for all $g_i, g_j \in g$
- Associativity. $(g_i \circ g_j) \circ g_k = g_i \circ (g_j \circ g_k)$ for all $g_i, g_j, g_k \in g$.
- Identity. Exists an identity element e such that, $g_i \circ e = e \circ g_i = g_i$, for every $g_i \in g$
- Inverse. For every element g_i exist and inverse element g_i^{-1} such that $g_i \circ g_i^{-1} = e$.

Now, the modern formalism of mathematical physics applied to gravitation and high energy physics is based in differential topology where an important concept is the idea of a **manifold**, that in a intuitive way is defined as a space that is locally Euclidean, i.e. the space look Educliean in a small scale everywhere, then a more complex structure is make it differentiable [35].

Definition 4. An n dimensional differentiable manifold M is a topological space provided of a family of open sets $\{U_i\}$ which covers M, so $M = \cup_i U_i$ with a collection of homeomorphisms $\{\phi_i\}$, $\phi_i : U_i \to U_i'$ with U_i' is an open subset of $\mathbb{R}^n$ and given U_i and U_j such that $U_i \cap U_j \neq \varnothing$, the map $\varphi_{ij} = \phi_i \circ \phi_j^{-1} : \phi_j(U_i \cap U_j) \to \phi_i(U_i \cap U_j)$ has an inverse and is infinitely differentiable,

this is a very important definition, because a lot of mathematical structures as tensors, spinors, differential forms, covariant derivatives, etc., are based in it. Finally, is possible to define a Lie group in the following way

Definition 5. A *Lie group* G is a differential manifold M with dimension n that parametrizes the group operations $g(x)$, with the multiplication operation $g(x) \circ g(y) = g(z)$ where the coordinates x, y and z are in M and z depends on x and y by $z = \phi(x, y)$. The map $\phi(x, y)$ and the inverse operation $g^{-1}(x)$ need to be differentiable for every $x, y \in M$.

An important example of Lie group es the $U(n)$ group, that is the *unitary group* which consists of the $n \times n$ complex matrices $A = (a_{ij})$ with $A^\dagger := \bar{A}^T = A^{-1}$. $A^\dagger$ denotes the *hermitian adjoint* of A and $SU(n)$ is the **special unitary group** that implies $\det A = 1$. The number of generators of the $SU(N)$ group is $n^2 - 1$.

Also, exist a direct connection between a Lie algebra an a Lie group, this connection is given by the exponential map (frankel):

Definition 6. The *exponential map* is the map $\exp : \mathfrak{g} \to G$, that sends $A \rightarrowtail e^A$, is a diffeomorphism (differentiable isomorphism between manifolds) of some neighborhood of $0 \in \mathfrak{g}$ to a neighborhood of $e \in G$.

Theorem 1. *Using the exponential map* $\det(e^A) = e^{trA}$

Remark. The past theorem implies that if a representation of the $SU(N)$ group the unitary $n \times n$ matrix with determinant equal to 0, then the generators (elements of the algebra) are $n \times n$ traceless matrices.

The direct application of all of these thing is in the gauge groups in the quantum gauge field theories. As the group associated group to the rotation invariance of the spin $SU(2)$, the flavor symmetry related to the $SU(N_f)$ group or the color charge with the $SU(N_c)$ group, in particular both have the physical value $N_f = N_c = 3$, so the general explanation of $SU(2)$ and $SU(3)$ is presented next:

SU(2)

It is a three parameter group and has a representation given by the 2×2 of the form

$$U(\alpha_1, \alpha_2, \alpha_3) = \exp\{i\alpha_i\sigma_i\}, \tag{A.8}$$

where σ_i are the Pauli matrices, that are 2×2 hermitian traceless matrices given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.9}$$

and the generators of the group are defined as $J_i = \sigma_i/2$ and satisfy the commutation relation

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \tag{A.10}$$

where ϵ_{ijk} is the Levi-Civita tensor, it is a totally antisymmetric tensor with $\epsilon_{123} = 1$. The $SU(2)$ group has a invariant element:

$$J^2 = J_1^2 + J_2^2 + J_3^2, \tag{A.11}$$

this operator is also called the *Casimir operator* of the algebra and it commutes with every generator J_i. Almost every Lie algebra has at least a Casimir operator [36].

SU(3)

This group is a group with eight parameters. Taking the representation of the 3×3 unitary matrices with determinant equal to cero

$$U(\alpha^1, ..., \alpha^8) = \exp\{i\alpha^a\lambda^a\} \tag{A.12}$$

here λ^a are the 3×3 traceless hermitian matrices also called Gell-Mann matrices and given by

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{A.13}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

these matrices have the normalization condition

$$tr(\lambda^a\lambda^b) = 2\delta^{ab} \tag{A.14}$$

and the generators $T^a = \frac{\lambda^a}{2}$satisfy the commutation relation

$$\left[T^a, T^b\right] = if^{abc}T^c, \tag{A.15}$$

where f^{abc} is the structure constant of the algebra and it is an totally antisymmteric tensor with nonvanishing values.

$$\begin{gathered} f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2}, \\ f^{156} = f^{367} = -\frac{1}{2}, \quad f^{678} = \frac{\sqrt{3}}{2}, \end{gathered} \tag{A.16}$$

also, the group satisfies the anticommutation relation

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab}\mathbb{1} + d^{abc}T^c, \tag{A.17}$$

d^{abc} is a totally symmetric tensor with nonvanishing values

$$\begin{gathered} d^{118} = d^{228} = d^{d338} = \frac{1}{\sqrt{3}}, \quad d^{448} = d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}} \\ d^{888} = -\frac{1}{\sqrt{3}}, \quad d^{146} = d^{157} = d^{256} = d^{344} = d^{355} = \frac{1}{2} \\ d^{247} = d^{366} = d^{337} = -\frac{1}{2}, \end{gathered} \tag{A.18}$$

the tensor f^{abc} can be computed by the generators as

$$f^{acb} = -2itr([T^a, T^b]T^c), \tag{A.19}$$

and for the tensor d^{abc}

$$d^{acb} = 2tr(\{T^a, T^b\}T^c), \tag{A.20}$$

[34][36].

Appendix B

Operator Reductions

The complete expansion in the operator basis of the commutator (5.1) of each operator in the N_c expansion for the axial current are listed bellow:

- Flavor singlet contribution:

$$[G^{ia},[G^{ia},G^{kc}]] = \frac{3N_f^2-4}{4N_f}G^{kc}, \tag{B.1}$$

$$\begin{aligned}&[G^{ia},[G^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ia},G^{kc}]] + [G^{ia},[\mathcal{D}_2^{ia},G^{kc}]] = \\ &\qquad -\frac{2(N_c+N_f)}{N_f}G^{kc} + \frac{9N_f^2+8N_f-4}{4N_f}\mathcal{D}_2^{kc},\end{aligned} \tag{B.2}$$

$$\begin{aligned}&[G^{ia},[G^{ia},\mathcal{D}_3^{kc}]] + [\mathcal{D}_3^{ia},[G^{ia},G^{kc}]] + [G^{ia},[\mathcal{D}_3^{ia},G^{kc}]] = -[N_c(N_c+2N_f)-2N_f+8]G^{kc} \\ &\qquad -3(N_c+N_f)\mathcal{D}_2^{kc} + \frac{13N_f^2+16N_f-12}{4N_f}\mathcal{D}_3^{kc} + \frac{N_f^2+2N_f-8}{N_f}\mathcal{O}_3^{kc}\end{aligned} \tag{B.3}$$

$$\begin{aligned}&[G^{ia},[G^{ia},\mathcal{O}_3^{kc}]] + [\mathcal{O}_3^{ia},[G^{ia},G^{kc}]] + [G^{ia},[\mathcal{O}_3^{ia},G^{kc}]] = -[N_c(N_c+2N_f)-N_f]G^{kc} \\ &\qquad -\frac{1}{2}(N_c+N_f)\mathcal{D}_2^{kc} + \frac{1}{2}(N_f+1)\mathcal{D}_3^{kc} + \frac{15N_f^2+12N_f-4}{4N_f}\mathcal{O}_3^{kc},\end{aligned} \tag{B.4}$$

$$\begin{aligned}&[G^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},G^{kc}]] = \\ &\qquad \frac{N_c(N_c+2N_f)(N_f-2)-6N_f^2}{2N_f}G^{kc} + \frac{2(N_c+N_f)(N_f-1)}{N_f}\mathcal{D}_2^{kc} \\ &\qquad + \frac{1}{4}(3N_f+2)\mathcal{D}_3^{kc} + \frac{1}{2}N_f\mathcal{O}_3^{kc},\end{aligned} \tag{B.5}$$

$$\begin{aligned}&[G^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_3^{kc}]] + [\mathcal{D}_2^{ia},[G^{ia},\mathcal{D}_3^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{D}_3^{ia},G^{kc}]] \\ &+ [\mathcal{D}_3^{ia},[\mathcal{D}_2^{ia},G^{kc}]] + [G^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_3^{ia},[G^{ia},\mathcal{D}_2^{kc}]] \\ &= -12(N_c+N_f)G^{kc} + [N_c(N_c+2N_f)-2N_f+8]\mathcal{D}_2^{kc} + \frac{(N_c+N_f)(7N_f-4)}{N_f}\mathcal{D}_3^{kc} \\ &\quad + \frac{2(N_c+N_f)(3N_f-4)}{N_f}\mathcal{O}_3^{kc} + \frac{3N_f^2-4N_f-4}{N_f}\mathcal{D}_4^{kc},\end{aligned} \tag{B.6}$$

$$\begin{aligned}
&[G^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[G^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},G^{kc}]] \\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},G^{kc}]]+[G^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[G^{ia},\mathcal{D}_2^{kc}]] \\
&=-\frac{3}{2}[N_c(N_c+2N_f)-8N_f]\mathcal{D}_2^{kc}-\frac{9}{2}(N_c+N_f)\mathcal{D}_3^{kc} \\
&\quad -\frac{2(N_c+N_f)}{N_f}\mathcal{O}_3^{kc}+(3N_f+10)\mathcal{D}_4^{kc},
\end{aligned} \tag{B.7}$$

$$[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]]=\frac{N_c(N_c+2N_f)(N_f-2)-2N_f^2}{2N_f}\mathcal{D}_2^{kc}+\frac{1}{2}(N_f+2)\mathcal{D}_4^{kc}, \tag{B.8}$$

$$\begin{aligned}
&[G^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[G^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ia},G^{kc}]] \\
&=-6[N_c(N_c+2N_f)+2N_f]G^{kc}+6(N_c+N_f)\mathcal{D}_2^{kc} \\
&\quad +3[N_c(N_c+2N_f)+2N_f-2]\mathcal{D}_3^{kc}+\frac{N_cN_f(N_c+2N_f)+12N_f(N_f-2)+8}{N_f}\mathcal{O}_3^{kc} \\
&\quad -3(N_c+N_f)\mathcal{D}_4^{kc}+\frac{3(N_f^2+2N_f-4)}{2N_f}\mathcal{D}_5^{kc}+\frac{N_f^2+8N_f-20}{N_f}\mathcal{O}_5^{kc},
\end{aligned} \tag{B.9}$$

$$\begin{aligned}
&[G^{ia},[\mathcal{O}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[G^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ia},G^{kc}]] \\
&=-\frac{3}{2}N_c(N_c+2N_f)G^{kc}-\frac{1}{4}[N_c(N_c+2N_f)-6N_f]\mathcal{D}_3^{kc} \\
&\quad -\frac{1}{4}[13N_c(N_c+2N_f)-38N_f-12]\mathcal{O}_3^{kc}-\frac{3}{4}(N_c+N_f)\mathcal{D}_4^{kc} \\
&\quad +\frac{1}{4}(N_f+3)\mathcal{D}_5^{kc}+\frac{1}{2}(11N_f+16)\mathcal{O}_5^{kc},
\end{aligned} \tag{B.10}$$

$$\begin{aligned}
&[G^{ia},[\mathcal{D}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[G^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ia},G^{kc}]] \\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ia},G^{kc}]]+[G^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{O}_3^{ia},[G^{ia},\mathcal{D}_3^{kc}]] \\
&=-24(N_c+N_f)\mathcal{D}_2^{kc}-3[2N_c(N_c+2N_f)-5N_f]\mathcal{D}_3^{kc}-[N_c(N_c+2N_f)-2N_f+8]\mathcal{O}_3^{kc} \\
&\quad -5(N_c+N_f)\mathcal{D}_4^{kc}+(5N_f+11)\mathcal{D}_5^{kc}+\frac{N_f^2+2N_f-8}{N_f}\mathcal{O}_5^{kc},
\end{aligned} \tag{B.11}$$

$$\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]] \\
&=\frac{N_c(N_c+2N_f)(N_f-2)-6N_f^2}{2N_f}\mathcal{D}_3^{kc} \\
&\quad +\frac{4(N_c+N_f)(N_f-1)}{N_f}\mathcal{D}_4^{kc}+\frac{1}{2}(3N_f+2)\mathcal{D}_5^{kc},
\end{aligned} \tag{B.12}$$

$$\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]] \\
&\qquad =\frac{N_c(N_c+2N_f)(N_f-2)-6N_f^2}{2N_f}\mathcal{O}_3^{kc}+\frac{1}{2}N_f\mathcal{O}_5^{kc},
\end{aligned} \tag{B.13}$$

$$
\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_2^{kc}]] \\
&= -6(N_c+N_f)\mathcal{D}_3^{kc}+[N_c(N_c+2N_f)-2N_f+8]\mathcal{D}_4^{kc} \\
&\quad +\frac{(N_c+N_f)(7N_f-4)}{N_f}\mathcal{D}_5^{kc}+\frac{3N_f^2-4N_f-4}{N_f}\mathcal{D}_6^{kc},
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]] \\
&\qquad = -12(N_c+N_f)\mathcal{O}_3^{kc}+\frac{2(N_c+N_f)(3N_f-4)}{N_f}\mathcal{O}_5^{kc},
\end{aligned} \tag{B.15}
$$

$$
\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]] \\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_3^{kc}]] \\
&= -\frac{3}{2}[N_c(N_c+2N_f)-8N_f]\mathcal{D}_2^{kc}-6(N_c+N_f)\mathcal{D}_3^{kc} \\
&\quad -\frac{1}{4}[5N_c(N_c+2N_f)-58N_f-48]\mathcal{D}_4^{kc} \\
&\quad -\frac{11}{4}(N_c+N_f)\mathcal{D}_5^{kc}+\frac{1}{2}(3N_f+14)\mathcal{D}_6^{kc},
\end{aligned} \tag{B.16}
$$

$$
\begin{aligned}
&[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_3^{kc}]] \\
&= -2[N_c(N_c+2N_f)+2N_f]\mathcal{D}_3^{kc}+4(N_c+N_f)\mathcal{D}_4^{kc} \\
&\quad +2[N_c(N_c+2N_f)+2N_f-2]\mathcal{D}_5^{kc}-2(N_c+N_f)\mathcal{D}_6^{kc}+\frac{N_f^2+2N_f-4}{N_f}\mathcal{D}_7^{kc},
\end{aligned} \tag{B.17}
$$

$$
\begin{aligned}
&[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_3^{kc}]] \\
&= -6[N_c(N_c+2N_f)+2N_f]\mathcal{O}_3^{kc}+\frac{[N_cN_f(N_c+2N_f)+12N_f^2-24N_f+8]}{N_f}\mathcal{O}_5^{kc} \\
&\quad +\frac{N_f^2+8N_f-20}{N_f}\mathcal{O}_7^{kc},
\end{aligned} \tag{B.18}
$$

$$
\begin{aligned}
[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_3^{kc}]] = &-24(N_c+N_f)\mathcal{D}_2^{kc}-\frac{3}{2}[5N_c(N_c+2N_f)-8N_f]\mathcal{D}_3^{kc} \\
&-26(N_c+N_f)\mathcal{D}_4^{kc}-2[2N_c(N_c+2N_f)-11N_f-6]\mathcal{D}_5^{kc} \\
&-\frac{7}{2}(N_c+N_f)\mathcal{D}_6^{kc}+\frac{1}{2}(5N_f+17)\mathcal{D}_7^{kc},
\end{aligned} \tag{B.19}
$$

$$
\begin{aligned}
[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ia},\mathcal{O}_3^{kc}]] = &-\frac{3}{2}N_c(N_c+2N_f)\mathcal{O}_3^{kc}-\frac{1}{4}[9N_c(N_c+2N_f)-34N_f-12]\mathcal{O}_5^{kc} \\
&+\frac{5}{2}(N_f+2)\mathcal{O}_7^{kc}.
\end{aligned} \tag{B.20}
$$

- Flavor octet contribution

$$d^{ab8}[G^{ia},[G^{ib},G^{kc}]] = \frac{3N_f^2-16}{8N_f}d^{c8e}G^{ke} + \frac{N_f^2-4}{2N_f^2}\delta^{c8}J^k, \tag{B.21}$$

$$\begin{aligned} & d^{ab8}([G^{ia},[G^{ib},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ib},G^{kc}]] + [G^{ia},[\mathcal{D}_2^{ib},G^{kc}]]) \\ & = -\frac{2(N_c+N_f)}{N_f}d^{c8e}G^{ke} + \frac{(N_c+N_f)(N_f-2)}{N_f^2}\delta^{c8}J^k + \frac{1}{8}(5N_f+8)d^{c8e}\mathcal{D}_2^{ke} \\ & \quad - \frac{2}{N_f}\{G^{kc},T^8\} + \frac{N_f^2+2N_f-4}{2N_f}\{G^{k8},T^c\} + \frac{N_f+2}{4}[J^2,[T^8,G^{kc}]], \end{aligned} \tag{B.22}$$

$$\begin{aligned} & d^{ab8}([G^{ia},[G^{ib},\mathcal{D}_3^{kc}]] + [\mathcal{D}_3^{ia},[G^{ib},G^{kc}]] + [G^{ia},[\mathcal{D}_3^{ib},G^{kc}]]) \\ & = (N_f-8)d^{c8e}G^{ke} - \frac{3N_c(N_c+2N_f)-8N_f+16}{2N_f}\delta^{c8}J^k - (N_c+N_f)\{G^{kc},T^8\}\mathcal{D}_2^{ke} \\ & \quad - \frac{3}{2}(N_c+N_f)d^{c8e} + \frac{3}{2}(N_c+N_f)[J^2,[T^8,G^{kc}]] + \frac{5N_f^2+12N_f-16}{8N_f}d^{c8e}\mathcal{D}_3^{ke} \\ & \quad + \frac{(N_f+6)(N_f-4)}{2N_f}d^{c8e}\mathcal{O}_3^{ke} + \frac{(N_f+4)(N_f-1)}{N_f^2}\delta^{c8}\{J^2,J^k\} \\ & \quad + \frac{(N_f+4)(N_f-1)}{N_f}\{G^{k8},\{J^r,G^{rc}\}\} - \frac{3}{4}\{J^k,\{T^c,T^8\}\} \\ & \quad + (N_f+1)\{J^k,\{G^{rc},G^{r8}\}\} + \frac{N_f-4}{N_f}\{G^{kc},\{J^r,G^{r8}\}\}, \end{aligned} \tag{B.23}$$

$$\begin{aligned} & d^{ab8}([G^{ia},[G^{ib},\mathcal{O}_3^{kc}]] + [\mathcal{O}_3^{ia},[G^{ib},G^{kc}]] + [G^{ia},[\mathcal{O}_3^{ib},G^{kc}]]) \\ & = \frac{N_f}{2}d^{c8e}G^{ke} - \frac{N_c(N_c+2N_f)}{4N_f}\delta^{c8}J^k - \frac{1}{4}(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke} - (N_c+N_f)\{G^{kc},T^8\} \\ & - (N_c+N_f)[J^2,[T^8,G^{kc}]] + \frac{N_f^2+N_f-8}{4N_f}d^{c8e}\mathcal{D}_3^{ke} + \frac{7N_f^2+8N_f-16}{8N_f}d^{c8e}\mathcal{O}_3^{ke} \\ & + (N_f+2)\{G^{kc},\{J^r,G^{r8}\}\} - \frac{1}{2}(N_f+2)\{G^{k8},\{J^r,G^{rc}\}\} - \frac{1}{8}\{J^k,\{T^c,T^8\}\} \\ & - \frac{N_f^2+N_f-8}{2N_f}\{J^k,\{G^{rc},G^{r8}\}\} + \frac{2N_f^2+N_f-8}{2N_f^2}\delta^{c8}\{J^2,J^k\}, \end{aligned} \tag{B.24}$$

$$\begin{aligned} & d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ib},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},G^{kc}]]) \\ & = -\frac{3}{2}N_f d^{c8e}G^{ke} + \frac{(N_c+N_f)(N_f-4)}{2N_f}\{G^{kc},T^8\} + \frac{(N_c+N_f)(N_f-2)}{N_f}\{G^{k8},T^c\} \\ & + \frac{1}{4}(N_c+N_f)[J^2,[T^8,G^{kc}]] + \frac{3}{8}N_f d^{c8e}\mathcal{D}_3^{ke} + \frac{1}{4}(N_f-2)d^{c8e}\mathcal{O}_3^{ke} \\ & + \frac{1}{2}\{G^{kc},\{J^r,G^{r8}\}\} + \frac{1}{2}\{G^{k8},\{J^r,G^{rc}\}\} + \frac{N_f-2}{2N_f}\{J^k,\{T^c,T^8\}\}, \end{aligned} \tag{B.25}$$

$$\begin{aligned}
&d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{ia},[G^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ib},G^{kc}]] \\
&+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ib},G^{kc}]]+[G^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{ia},[G^{ib},\mathcal{D}_2^{kc}]]) \\
&=-6(N_c+N_f)d^{c8e}G^{ke}-3(N_f-2)d^{c8e}\mathcal{D}_2^{ke}-6\{G^{kc},T^8\}+2(N_f+1)\{G^{k8},T^c\} \\
&\quad+\frac{N_f^2+8}{N_f}[J^2,[T^8,G^{kc}]]+\frac{3}{2}(N_c+N_f)d^{c8e}\mathcal{D}_3^{ke}+\frac{(N_c+N_f)(N_f-4)}{N_f}d^{c8e}\mathcal{O}_3^{ke} \\
&\quad+\frac{2(N_c+N_f)(N_f-2)}{N_f}\{G^{kc},\{J^r,G^{r8}\}\}+\frac{2(N_c+N_f)(N_f-2)}{N_f}\{G^{k8},\{J^r,G^{rc}\}\} \\
&\quad+\frac{N_c+N_f}{2}\{J^k,\{T^c,T^8\}\}+\frac{3}{2}(N_f-2)d^{c8e}\mathcal{D}_4^{ke}-(N_f+4)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} \\
&\quad+4\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}+\frac{3N_f-8}{N_f}\{J^2,\{G^{kc},T^8\}\}+\frac{N_f^2+3N_f-8}{N_f}\{J^2,\{G^{k8},T^c\}\} \\
&\quad+\frac{N_f^2+4N_f-8}{2N_f}\{J^2,[J^2,[T^8,G^{kc}]]\}-\frac{1}{8}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
&\quad+\frac{1}{8}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}-\frac{1}{8}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}+\frac{1}{8}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
&\quad+\frac{1}{8}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\},
\end{aligned} \tag{B.26}$$

$$\begin{aligned}
&d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[G^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ib},G^{kc}]] \\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ib},G^{kc}]]+[G^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[G^{ib},\mathcal{D}_2^{kc}]]) \\
&=6N_fd^{c8e}\mathcal{D}_2^{ke}-\frac{(N_c+N_f)(5N_f+8)}{4N_f}d^{c8e}\mathcal{D}_3^{ke}-\frac{2(N_c+N_f)}{N_f}d^{c8e}\mathcal{O}_3^{ke} \\
&-\frac{3}{4}(N_c+N_f)\{J^k,\{T^c,T^8\}\}-\frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^k,\{G^{rc},G^{r8}\}\} \\
&+\frac{2(N_c+N_f)(N_f-2)}{N_f^2}\delta^{c8}\{J^2,J^k\}+\frac{1}{2}(N_f+9)d^{c8e}\mathcal{D}_4^{ke}-\frac{2}{N_f}\{J^2,\{G^{kc},T^8\}\} \\
&+\frac{N_f^2+9N_f+4}{2N_f}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}-\frac{9N_f-4}{2N_f}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
&+\frac{N_f^2+2N_f-4}{2N_f}\{J^2,\{G^{k8},T^c\}\}+\frac{1}{4}(N_f+2)\{J^2,[J^2,[T^8,G^{kc}]]\} \\
&-\frac{15}{32}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}+\frac{15}{32}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}-\frac{15}{32}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&+\frac{15}{32}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}+\frac{15}{32}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\},
\end{aligned} \tag{B.27}$$

$$\begin{aligned}
d^{ab8}[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]]=&-\frac{N_f}{2}d^{c8e}\mathcal{D}_2^{ke}+\frac{(N_c+N_f)(N_f-4)}{4N_f}\{J^k,\{T^c,T^8\}\} \\
&+\frac{N_f}{4}d^{c8e}\mathcal{D}_4^{ke}+\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\},
\end{aligned} \tag{B.28}$$

$$
\begin{aligned}
&d^{ab8}([G^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[G^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ib},G^{kc}]]) \\
&=-6N_f d^{c8e}G^{ke}+\frac{3N_c(N_c+2N_f)}{N_f}\delta^{c8}J^k+3(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke}-6(N_c+N_f)\{G^{kc},T^8\} \\
&+2(N_c+N_f)[J^2,[T^8,G^{kc}]]-3d^{c8e}\mathcal{D}_3^{ke}+(N_f-14)d^{c8e}\mathcal{O}_3^{ke} \\
&+\frac{5N_f^2-10N_f+16}{N_f}\{G^{kc},\{J^r,G^{r8}\}\}+\frac{7N_f^2-2N_f-16}{N_f}\{G^{k8},\{J^r,G^{rc}\}\} \\
&+\frac{3}{2}\{J^k,\{T^c,T^8\}\}-6(N_f-1)\{J^k,\{G^{rc},G^{r8}\}\}-\frac{3[N_c(N_c+2N_f)+4]}{2N_f}\delta^{c8}\{J^2,J^k\} \\
&-\frac{3}{2}(N_c+N_f)d^{c8e}\mathcal{D}_4^{ke}+5(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}+(N_c+N_f)\{J^2,\{G^{kc},T^8\}\} \\
&+2(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\}+\frac{9N_f-8}{8N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
&-\frac{9N_f-8}{8N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}+\frac{9N_f-8}{8N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&-\frac{9N_f-8}{8N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}-\frac{9N_f-8}{8N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} \\
&+\frac{3}{4}(N_f+2)d^{c8e}\mathcal{D}_5^{ke}+\frac{(N_f+8)(N_f-4)}{2N_f}d^{c8e}\mathcal{O}_5^{ke}-\frac{3}{4}\{J^2,\{J^k,\{T^c,T^8\}\}\} \\
&+\frac{6(N_f-4)}{N_f}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}+\frac{N_f^2+6N_f-8}{N_f}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
&+3(N_f-1)\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\}-\frac{2N_f^2+3N_f-4}{N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} \\
&+\frac{3}{N_f}\delta^{c8}\{J^2,\{J^2,J^k\}\},
\end{aligned}
\tag{B.29}
$$

$$
\begin{aligned}
&d^{ab8}([G^{ia},[\mathcal{O}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[G^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ib},G^{kc}]]) \\
&=-\frac{3}{2}(N_c+N_f)\{G^{kc},T^8\}-2(N_c+N_f)[J^2,[T^8,G^{kc}]]+\frac{N_f^2-4}{2N_f}d^{c8e}\mathcal{D}_3^{ke} \\
&+\frac{1}{2}(4N_f+3)d^{c8e}\mathcal{O}_3^{ke}+\frac{1}{4}(11N_f+6)\{G^{kc},\{J^r,G^{r8}\}\}-\frac{1}{4}(5N_f+6)\{G^{k8},\{J^r,G^{rc}\}\} \\
&-\frac{N_f^2-4}{N_f}\{J^k,\{G^{rc},G^{r8}\}\}-\frac{3N_cN_f(N_c+2N_f)-8N_f^2+32}{8N_f^2}\delta^{c8}\{J^2,J^k\} \\
&-\frac{3}{8}(N_c+N_f)d^{c8e}\mathcal{D}_4^{ke}+\frac{11}{4}(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
&-\frac{13}{4}(N_c+N_f)\{J^2,\{G^{kc},T^8\}\}-\frac{7}{4}(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\} \\
&-\frac{3[N_cN_f+6N_f^2+10N_f-32]}{64N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
&+\frac{3[N_cN_f+6N_f^2+10N_f-32]}{64N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
&-\frac{3[N_cN_f+6N_f^2+10N_f-32]}{64N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&+\frac{3[N_cN_f+6N_f^2+10N_f-32]}{64N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
&+\frac{3[N_cN_f+6N_f^2+10N_f-32]}{64N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} \\
&+\frac{N_f^2+3N_f-8}{8N_f}d^{c8e}\mathcal{D}_5^{ke}+\frac{1}{4}(3N_f+8)d^{c8e}\mathcal{O}_5^{ke} \\
&+2(N_f+3)\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}-\frac{1}{4}(3N_f+8)\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
&-\frac{3}{16}\{J^2,\{J^k,\{T^c,T^8\}\}\}-\frac{N_f^2+3N_f-16}{4N_f}\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\} \\
&-\frac{2N_f^2+5N_f+4}{4N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}+\frac{2N_f^2+3N_f-8}{4N_f^2}\delta^{c8}\{J^2,\{J^2,J^k\}\},
\end{aligned}
\tag{B.30}
$$

$$
\begin{aligned}
&d^{ab8}([G^{ia},[\mathcal{D}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[G^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ib},G^{kc}]]\\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ib},G^{kc}]]+[G^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{O}_3^{ia},[G^{ib},\mathcal{D}_3^{kc}]])\\
&=-\frac{12N_c(N_c+2N_f)}{N_f}\delta^{c8}J^k-12(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke}+\frac{1}{2}(7N_f-8)d^{c8e}\mathcal{D}_3^{ke}\\
&+(N_f-8)d^{c8e}\mathcal{O}_3^{ke}-6\{J^k,\{T^c,T^8\}\}+8(N_f+1)\{J^k,\{G^{rc},G^{r8}\}\}\\
&-\frac{5N_c(N_c+2N_f)-32N_f+16}{2N_f}\delta^{c8}\{J^2,J^k\}-\frac{5}{2}(N_c+N_f)d^{c8e}\mathcal{D}_4^{ke}\\
&-11(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}-(N_c+N_f)\{J^2,\{G^{kc},T^8\}\}\\
&+\frac{3}{2}(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\}\\
&+\frac{3N_cN_f+18N_f^2-12N_f-116}{32N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\\
&-\frac{3N_cN_f+18N_f^2-12N_f-116}{32N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\\
&+\frac{3N_cN_f+18N_f^2-12N_f-116}{32N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\\
&-\frac{3N_cN_f+18N_f^2-12N_f-116}{32N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\\
&-\frac{3N_cN_f+18N_f^2-12N_f-116}{32N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\\
&+\frac{N_f^2+5N_f-8}{2N_f}d^{c8e}\mathcal{D}_5^{ke}+\frac{(N_f+6)(N_f-4)}{2N_f}d^{c8e}\mathcal{O}_5^{ke}\\
&+\frac{N_f-4}{N_f}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}+\frac{(N_f+4)(N_f-1)}{N_f}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}\\
&-\frac{5}{4}\{J^2,\{J^k,\{T^c,T^8\}\}\}-\frac{N_f^2-3N_f-8}{N_f}\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\}\\
&+\frac{2N_f^2+5N_f+4}{N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}+\frac{2N_f^2+5N_f-8}{N_f^2}\delta^{c8}\{J^2,\{J^2,J^k\}\},
\end{aligned}
\tag{B.31}
$$

$$
\begin{aligned}
&d^{ab8}([\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]]\\
&=-\frac{3}{2}N_fd^{c8e}\mathcal{D}_3^{ke}+\frac{2(N_c+N_f)(N_f-2)}{N_f}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\\
&+\frac{(N_c+N_f)(N_f-4)}{N_f}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}+\frac{1}{4}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\\
&-\frac{1}{4}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}+\frac{1}{4}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}-\frac{1}{4}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\\
&-\frac{1}{4}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}+\frac{3}{4}N_fd^{c8e}\mathcal{D}_5^{ke}+\frac{N_f-2}{N_f}\{J^2,\{J^k,\{T^c,T^8\}\}\}\\
&+\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\},
\end{aligned}
\tag{B.32}
$$

$$
\begin{aligned}
&d^{ab8}([\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]]) \\
&=-\frac{3}{2}N_f d^{c8e}\mathcal{O}_3^{ke}-\frac{(N_c+N_f)(N_f-2)}{N_f}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} \\
&-\frac{(N_c+N_f)(N_f-4)}{2N_f}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}+\frac{(N_c+N_f)(N_f-4)}{2N_f}\{J^2,\{G^{kc},T^8\}\} \\
&+\frac{(N_c+N_f)(N_f-2)}{N_f}\{J^2,\{G^{k8},T^c\}\}+\frac{1}{4}(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\} \\
&-\frac{7}{16}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}+\frac{7}{16}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}-\frac{7}{16}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&+\frac{7}{16}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}+\frac{7}{16}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}+\frac{1}{4}(N_f-2)d^{c8e}\mathcal{O}_5^{ke} \\
&+\frac{1}{2}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}+\frac{1}{2}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
&-\frac{1}{2}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\},
\end{aligned}
\tag{B.33}
$$

$$
\begin{aligned}
&d^{ab8}([\mathcal{D}_2^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_2^{kc}]]) \\
&=-3(N_c+N_f)d^{c8e}\mathcal{D}_3^{ke}-3(N_f-2)d^{c8e}\mathcal{D}_4^{ke} \\
&+2(N_f+1)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}-6\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
&+\frac{N_c(3N_f-4)+4N_f^2-6N_f-4}{8N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
&-\frac{N_c(3N_f-4)+4N_f^2-6N_f-4}{8N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
&+\frac{N_c(3N_f-4)+4N_f^2-6N_f-4}{8N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&-\frac{N_c(3N_f-4)+4N_f^2-6N_f-4}{8N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
&-\frac{N_c(3N_f-4)+4N_f^2-6N_f-4}{8N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} \\
&+\frac{3}{2}(N_c+N_f)d^{c8e}\mathcal{D}_5^{ke}+\frac{1}{2}(N_c+N_f)\{J^2,\{J^k,\{T^c,T^8\}\}\} \\
&+\frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}+\frac{3}{2}(N_f-2)d^{c8e}\mathcal{D}_6^{ke} \\
&-\frac{N_f+8}{N_f}\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\}+\frac{7N_f-8}{N_f}\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\},
\end{aligned}
\tag{B.34}
$$

$$
\begin{aligned}
& d^{ab8}([\mathcal{D}_2^{ia},[\mathcal{O}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_2^{kc}]]) \\
& = -6(N_c+N_f)d^{c8e}\mathcal{O}_3^{ke} - 2(N_f+1)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\} + 6\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
& - 6\{J^2,\{G^{kc},T^8\}\} + 2(N_f+1)\{J^2,\{G^{k8},T^c\}\} + \frac{N_f^2+8}{N_f}\{J^2,[J^2,[T^8,G^{kc}]]\} \\
& - \frac{N_c(29N_f-28)+42N_f^2-116N_f-16}{32N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
& + \frac{N_c(29N_f-28)+42N_f^2-116N_f-16}{32N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
& - \frac{N_c(29N_f-28)+42N_f^2-116N_f-16}{32N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
& + \frac{N_c(29N_f-28)+42N_f^2-116N_f-16}{32N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
& + \frac{N_c(29N_f-28)+42N_f^2-116N_f-16}{32N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} \\
& + \frac{(N_c+N_f)(N_f-4)}{N_f}d^{c8e}\mathcal{O}_5^{ke} + \frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\} \\
& + \frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
& - \frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} \\
& - \frac{N_f^2+3N_f-8}{N_f}\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\} - \frac{3N_f-8}{N_f}\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\} \\
& + \frac{3N_f-8}{N_f}\{J^2,\{J^2,\{G^{kc},T^8\}\}\} + \frac{N_f^2+3N_f-8}{N_f}\{J^2,\{J^2,\{G^{k8},T^c\}\}\} \\
& + \frac{N_f^2+4N_f-8}{2N_f}\{J^2,\{J^2,[J^2,[T^8,G^{kc}]]\}\} - \frac{1}{4}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} \\
& + \frac{1}{4}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} - \frac{1}{4}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} \\
& + \frac{1}{4}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} + \frac{1}{4}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\},
\end{aligned}
\tag{B.35}
$$

$$
\begin{aligned}
&d^{ab8}([\mathcal{D}_2^{ia},[\mathcal{D}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_2^{kc}]]\\
&+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_3^{kc}]])\\
&=6N_f d^{c8e}\mathcal{D}_2^{ke}-\frac{2(N_f+1)(N_c+N_f)}{N_f}d^{c8e}\mathcal{D}_3^{ke}-\frac{3}{4}(N_c+N_f)\{J^k,\{T^c,T^8\}\}\\
&-\frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^k,\{G^{rc},G^{r8}\}\}+\frac{2(N_c+N_f)(N_f-2)}{N_f^2}\delta^{c8}\{J^2,J^k\}\\
&+\frac{1}{4}(23N_f+24)d^{c8e}\mathcal{D}_4^{ke}+\frac{3}{2}(N_f+4)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}-6\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\\
&+\frac{N_c(49N_f+72)-12N_f^2-215N_f-24}{64N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\\
&-\frac{N_c(49N_f+72)-12N_f^2-215N_f-24}{64N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\\
&+\frac{N_c(49N_f+72)-12N_f^2-215N_f-24}{64N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\\
&-\frac{N_c(49N_f+72)-12N_f^2-215N_f-24}{64N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\\
&-\frac{N_c(49N_f+72)-12N_f^2-215N_f-24}{64N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\\
&-\frac{(N_c+N_f)(7N_f+8)}{8N_f}d^{c8e}\mathcal{D}_5^{ke}-\frac{5}{8}(N_c+N_f)\{J^2,\{J^k,\{T^c,T^8\}\}\}\\
&-\frac{2(N_c+N_f)(N_f-2)}{N_f}\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\}\\
&+\frac{(N_c+N_f)(N_f-2)}{2N_f}\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}\\
&+\frac{(N_c+N_f)(N_f-2)}{N_f^2}\delta^{c8}\{J^2,\{J^2,J^k\}\}+\frac{1}{4}(N_f+11)d^{c8e}\mathcal{D}_6^{ke}\\
&+\frac{1}{4}(2N_f+17)\{J^2,\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}\}-\frac{11}{4}\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\}\\
&-\frac{11}{32}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\}+\frac{11}{32}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\}\\
&-\frac{11}{32}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\}+\frac{11}{32}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\}\\
&+\frac{11}{32}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\},
\end{aligned}
\tag{B.36}
$$

$$
\begin{aligned}
& d^{ab8}([\mathcal{D}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_3^{kc}]]) \\
& = -2N_f d^{c8e}\mathcal{D}_3^{ke} + \frac{2N_c(N_c+2N_f)}{N_f}\delta^{c8}\{J^2,J^k\} \\
& + 2(N_c+N_f)d^{c8e}\mathcal{D}_4^{ke} - 4(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
& - \frac{N_c(4N_cN_f+24N_f^2+85N_f-104)-116N_f^2-432N_f+552}{16N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
& + \frac{N_c(4N_cN_f+24N_f^2+85N_f-104)-116N_f^2-432N_f+552}{16N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
& - \frac{N_c(4N_cN_f+24N_f^2+85N_f-104)-116N_f^2-432N_f+552}{16N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
& + \frac{N_c(4N_cN_f+24N_f^2+85N_f-104)-116N_f^2-432N_f+552}{16N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
& + \frac{N_c(4N_cN_f+24N_f^2+85N_f-104)-116N_f^2-432N_f+552}{16N_f}\times \\
& \{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} - 2d^{c8e}\mathcal{D}_5^{ke} + \{J^2,\{J^k,\{T^c,T^8\}\}\} \qquad \text{(B.37)} \\
& - 4(N_f-1)\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\} + 4(N_f-1)\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} \\
& - \frac{N_c(N_c+2N_f)+4}{N_f}\delta^{c8}\{J^2,\{J^2,J^k\}\} - (N_c+N_f)d^{c8e}\mathcal{D}_6^{ke} \\
& + 4(N_c+N_f)\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\} + \frac{3N_cN_f-8}{8N_f}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} \\
& - \frac{3N_cN_f-8}{8N_f}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} + \frac{3N_cN_f-8}{8N_f}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} \\
& - \frac{3N_cN_f-8}{8N_f}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} \\
& - \frac{3N_cN_f-8}{8N_f}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\} + \frac{1}{2}(N_f+2)d^{c8e}\mathcal{D}_7^{ke} \\
& - \frac{1}{2}\{J^2,\{J^2,\{J^k,\{T^c,T^8\}\}\}\} + 2(N_f-1)\{J^2,\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\}\} \\
& - \frac{N_f^2-2N_f+8}{N_f}\{J^2,\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}\} + \frac{2}{N_f}\delta^{c8}\{J^2,\{J^2,\{J^2,J^k\}\}\},
\end{aligned}
$$

$$
\begin{aligned}
& d^{ab8}([\mathcal{D}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_3^{kc}]]) \\
& = -6N_f d^{c8e}\mathcal{O}_3^{ke} + 6(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
& - 6(N_c+N_f)\{J^2,\{G^{kc},T^8\}\} + 2(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\} \\
& + \frac{26N_c^2N_f+48N_cN_f^2+949N_f-337N_cN_f+728N_c-194N_f^2-1972}{32N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
& - \frac{26N_c^2N_f+48N_cN_f^2+949N_f-337N_cN_f+728N_c-194N_f^2-1972}{32N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
& + \frac{26N_c^2N_f+48N_cN_f^2+949N_f-337N_cN_f+728N_c-194N_f^2-1972}{32N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
& - \frac{26N_c^2N_f+48N_cN_f^2+949N_f-337N_cN_f+728N_c-194N_f^2-1972}{32N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
& - \frac{26N_c^2N_f+48N_cN_f^2+949N_f-337N_cN_f+728N_c-194N_f^2-1972}{32N_f}\times \\
& \{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} + (N_f-14)d^{c8e}\mathcal{O}_5^{ke} + \frac{5N_f^2-10N_f+16}{N_f}\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\} \\
& + \frac{7N_f^2-2N_f-16}{N_f}\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
& - 6(N_f-1)\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} - (N_c+N_f)\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\} \\
& + (N_c+N_f)\{J^2,\{J^2,\{G^{kc},T^8\}\}\} + 2(N_c+N_f)\{J^2,\{J^2,[J^2,[T^8,G^{kc}]]\}\} \\
& - \frac{6N_cN_f+2N_f^2-33N_f-44}{16N_f}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} \\
& + \frac{6N_cN_f+2N_f^2-33N_f-44}{16N_f}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} \\
& - \frac{6N_cN_f+2N_f^2-33N_f-44}{16N_f}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} \\
& + \frac{6N_cN_f+2N_f^2-33N_f-44}{16N_f}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} \\
& + \frac{6N_cN_f+2N_f^2-33N_f-44}{16N_f}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\} \\
& + \frac{(N_f+8)(N_f-4)}{2N_f}d^{c8e}\mathcal{O}_7^{ke} + \frac{6(N_f-4)}{N_f}\{J^2,\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}\} \\
& + \frac{N_f^2+6N_f-8}{N_f}\{J^2,\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}\} \\
& - \frac{N_f^2+12N_f-32}{2N_f}\{J^2,\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}\},
\end{aligned}
\tag{B.38}
$$

$$
\begin{aligned}
& d^{ab8}[\mathcal{D}_3^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_3^{kc}]] = \\
& -\frac{12N_c(N_c+2N_f)}{N_f}\delta^{c8}J^k - 12(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke} + 2(N_f-2)d^{c8e}\mathcal{D}_3^{ke} - 6\{J^k,\{T^c,T^8\}\} \\
& + 8(N_f+1)\{J^k,\{G^{rc},G^{r8}\}\} - \frac{13N_c(N_c+2N_f)-16N_f+8}{N_f}\delta^{c8}\{J^2,J^k\} \\
& - 13(N_c+N_f)d^{c8e}\mathcal{D}_4^{ke} - 15(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} \\
& + \frac{N_c(69N_cN_f-40N_f^2-695N_f-1032)+222N_f^2+1983N_f+4092}{64N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
& - \frac{N_c(69N_cN_f-40N_f^2-695N_f-1032)+222N_f^2+1983N_f+4092}{64N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
& + \frac{N_c(69N_cN_f-40N_f^2-695N_f-1032)+222N_f^2+1983N_f+4092}{64N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
& - \frac{N_c(69N_cN_f-40N_f^2-695N_f-1032)+222N_f^2+1983N_f+4092}{64N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
& - \frac{N_c(69N_cN_f-40N_f^2-695N_f-1032)+222N_f^2+1983N_f+4092}{64N_f}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} \\
& + \frac{7N_f^2+2N_f-8}{2N_f}d^{c8e}\mathcal{D}_5^{ke} - \frac{13}{2}\{J^2,\{J^k,\{T^c,T^8\}\}\} + \frac{2(2N_f^2+7N_f+4)}{N_f}\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\} \\
& + \frac{1}{2}(11N_f+8)\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} \\
& - \frac{7N_cN_f(N_c+2N_f)-56N_f^2-8N_f+32}{4N_f^2}\delta^{c8}\{J^2,\{J^2,J^k\}\} \\
& - \frac{7}{4}(N_c+N_f)d^{c8e}\mathcal{D}_6^{ke} - 8(N_c+N_f)\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\} \\
& + \frac{6N_cN_f+22N_f^2-5N_f-116}{32N_f}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} \\
& - \frac{6N_cN_f+22N_f^2-5N_f-116}{32N_f}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} \\
& + \frac{6N_cN_f+22N_f^2-5N_f-116}{32N_f}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} \\
& - \frac{6N_cN_f+22N_f^2-5N_f-116}{32N_f}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} \\
& - \frac{6N_cN_f+22N_f^2-5N_f-116}{32N_f}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\} \\
& + \frac{(N_f+8)(N_f-1)}{4N_f}d^{c8e}\mathcal{D}_7^{ke} - \frac{7}{8}\{J^2,\{J^2,\{J^k,\{T^c,T^8\}\}\}\} \\
& - \frac{N_f^2-N_f-16}{2N_f}\{J^2,\{J^2,\{J^k,\{G^{rc},G^{r8}\}\}\}\} \\
& + \frac{5N_f^2+26N_f-8}{4N_f}\{J^2,\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}\} \\
& + \frac{2N_f^2+7N_f-8}{2N_f^2}\delta^{c8}\{J^2,\{J^2,\{J^2,J^k\}\}\},
\end{aligned}
\tag{B.39}
$$

$$
\begin{aligned}
&d^{ab8}[\mathcal{O}_3^{ia},[\mathcal{O}_3^{ib},\mathcal{O}_3^{kc}]] = \\
&\frac{3}{2}(N_c+N_f)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\} - \frac{3}{2}(N_c+N_f)\{J^2,\{G^{kc},T^8\}\} - 2(N_c+N_f)\{J^2,[J^2,[T^8,G^{kc}]]\} \\
&+ \frac{N_cN_f(-91N_c+1433+16N_f)+560N_c-4860N_f-184N_f^2-2340}{128N_f}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\} \\
&- \frac{N_cN_f(-91N_c+1433+16N_f)+560N_c-4860N_f-184N_f^2-2340}{128N_f}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\} \\
&+ \frac{N_cN_f(-91N_c+1433+16N_f)+560N_c-4860N_f-184N_f^2-2340}{128N_f}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\} \\
&- \frac{N_cN_f(-91N_c+1433+16N_f)+560N_c-4860N_f-184N_f^2-2340}{128N_f}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\} \\
&- \frac{N_cN_f(-91N_c+1433+16N_f)+560N_c-4860N_f-184N_f^2-2340}{128N_f}\times \\
&\quad \{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\} + \frac{3}{2}(N_f+1)d^{c8e}\mathcal{O}_5^{ke} \\
&+ \frac{1}{4}(11N_f+6)\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\} - \frac{1}{4}(5N_f+6)\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\} \\
&- \frac{3}{4}N_f\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\} + \frac{9}{4}(N_c+N_f)\{J^2,\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\} \\
&- \frac{9}{4}(N_c+N_f)\{J^2,\{J^2,\{G^{kc},T^8\}\}\} - \frac{3}{4}(N_c+N_f)\{J^2,\{J^2,[J^2,[T^8,G^{kc}]]\}\} \\
&- \frac{3N_cN_f+20N_f^2+52N_f-44}{64N_f}\{J^2,\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}\} \\
&+ \frac{3N_cN_f+20N_f^2+52N_f-44}{64N_f}\{J^2,\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}\} \\
&- \frac{3N_cN_f+20N_f^2+52N_f-44}{64N_f}\{J^2,\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}\} \\
&+ \frac{3N_cN_f+20N_f^2+52N_f-44}{64N_f}\{J^2,\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}\} \\
&+ \frac{3N_cN_f+20N_f^2+52N_f-44}{64N_f}\{J^2,\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}\} \\
&+ \frac{1}{4}(N_f+4)d^{c8e}\mathcal{O}_7^{ke} + (N_f+4)\{J^2,\{J^2,\{G^{kc},\{J^r,G^{r8}\}\}\}\} \\
&- \frac{1}{4}(N_f+4)\{J^2,\{J^2,\{G^{k8},\{J^r,G^{rc}\}\}\}\} \\
&- \frac{3}{8}(N_f+4)\{J^2,\{J^k,\{\{J^r,G^{rc}\},\{J^m,G^{m8}\}\}\}\},
\end{aligned}
\tag{B.40}
$$

- Flavor **27** contribution

$$
[G^{i8},[G^{i8},G^{kc}]] = \frac{1}{2}d^{8eg}d^{c8e}G^{kg} + \frac{1}{4}f^{8eg}f^{c8e}G^{kg} + \frac{1}{2N_f}d^{c88}J^k + \frac{1}{N_f}\delta^{c8}G^{k8}, \tag{B.41}
$$

$$\begin{aligned}
&[G^{i8},[G^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},G^{kc}]]+[G^{i8},[\mathcal{D}_2^{i8},G^{kc}]]= \\
&\qquad -d^{c8e}\{G^{ke},T^8\}+\frac{1}{2}d^{88e}\{G^{ke},T^c\}+if^{c8e}[G^{k8},\{J^r,G^{re}\}] \\
&\qquad +\frac{1}{2}d^{ceg}d^{88e}\mathcal{D}_2^{kg}+\frac{1}{2}d^{c8e}d^{8eg}\mathcal{D}_2^{kg}+\frac{9}{4}f^{c8e}f^{c8e}\mathcal{D}_2^{kg}+\frac{3}{N_f}\delta^{c8}\mathcal{D}_2^{k8},
\end{aligned} \tag{B.42}$$

$$\begin{aligned}
&[G^{i8},[G^{i8},\mathcal{D}_3^{kc}]]+[\mathcal{D}_3^{i8},[G^{i8},G^{kc}]]+[G^{i8},[\mathcal{D}_3^{i8},G^{kc}]]=-2\{G^{kc},\{G^{r8},G^{k8}\}\} \\
&+2\{G^{k8},\{G^{rc},G^{k8}\}\}+4d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\}-d^{c8e}\{G^{k8},\{J^r,G^{re}\}\} \\
&-3d^{c83}\{J^k,\{G^{re},G^{r8}\}\}+\frac{1}{N_f}d^{c88}\{J2,J^k\}+d^{88e}\{G^{ke},\{J^r,G^{rc}\}\} \\
&+d^{88e}\{J^k,\{G^{rc},G^{re}\}\}-\frac{1}{2}\epsilon^{kim}f^{c8e}\{T^e,\{J^i,G^{m8}\}\}-d^{ceg}d^{88e}\mathcal{D}_3^{kg}+\frac{3}{2}d^{c8e}d^{8eg}\mathcal{D}_3^{kg} \\
&+\frac{7}{4}f^{c8e}f^{8eg}\mathcal{D}_3^{kg}-\frac{3}{2}f^{c8e}f^{8eg}G^{kg}+d^{c8e}\mathcal{O}_3^{kg}+\frac{3}{N_f}\delta^{c8}\mathcal{D}_3^{k8}
\end{aligned} \tag{B.43}$$

$$\begin{aligned}
&[G^{i8},[G^{i8},\mathcal{O}_3^{kc}]]+[\mathcal{O}_3^{i8},[G^{i8},G^{kc}]]+[G^{i8},[\mathcal{O}_3^{i8},G^{kc}]]=-\{G^{kc},\{G^{r8,G^{r8}}\}\} \\
&-\{G^{k8},\{G^{rc},G^{r8}\}\}-d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\}+\frac{3}{2}d^{c8e}\{G^{k8},\{J^r,G^{re}\}\} \\
&-\frac{1}{2}d^{c8e}\{J^k,\{G^{re},G^{r8}\}\}+\frac{1}{N_f}d^{c88}\{J2,J^k\}+d^{88e}\{G^{kc},\{J^r,G^{re}\}\} \\
&-\frac{1}{2}d^{88e}\{G^{ke},\{J^r,G^{rc}\}\}-\frac{1}{2}d^{88e}\{J^k,\{G^{rc},G^{re}\}\}+\frac{3}{4}\epsilon^{kim}f^{c8e}\{T^e,\{J^i,G^{m8}\}\} \\
&+\frac{1}{2}d^{c8e}d^{8eg}\mathcal{D}_3^{kg}+\frac{3}{4}f^{c8e}f^{8eg}G^{kg}-d^{ceg}d^{88e}\mathcal{O}_3^{kg}+2d^{c8e}d^{8eg}\mathcal{O}+\frac{7}{4}f^{c8e}f^{8eg}\mathcal{O}_3^{kg} \\
&+\frac{1}{N_f}\delta^{c8}\mathcal{D}_3^{k8}+\frac{7}{N_f}\delta^{c8}\mathcal{O}_3^{k8},
\end{aligned} \tag{B.44}$$

$$\begin{aligned}
&[G^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{D}_2^{i8},G^{kc}]]=\frac{1}{2}\{G^{kc,\{T^8,T^8\}}\}+\{G^{k8},\{T^c,T8\}\} \\
&-\frac{1}{2}\epsilon^{kim}f^{c8e}\{T^e,\{T^c,G^{m8}\}\}+\frac{3}{4}f^{c8e}f^{8eg}\mathcal{D}_3^{kg}-2f^{c8e}f^{8eg}G^{kg}+\frac{1}{2}f^{c8e}f^{8eg}\mathcal{O}_3^{kg},
\end{aligned} \tag{B.45}$$

$$\begin{aligned}
&[G^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{D}_3^{i8},G^{kc}]] \\
&+[\mathcal{D}_3^{i8},[\mathcal{D}_2^{i8},G^{kc}]]+[G^{i8},[\mathcal{D}_3^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{i8},[G^{i8},\mathcal{D}_2^{kc}]] \\
&=2\{\{J^r,G^{rc}\},\{G^{k8},T^8\}\}+2\{\{J^r,G^{r8}\},\{G^{kc},T^8\}\}+2\{\{J^r,G^{r8}\},\{G^{k8},T^c\}\} \\
&+2d^{c8e}\{J^2,\{G^{ke},T^8\}\}-2d^{c8e}\{\mathcal{D}_2^{k8},\{J6r,G^{re}\}\}+d^{88e}\{J^2,\{G^{ke},T^c\}\} \\
&-d^{88e}\{\mathcal{D}_2^{kc},\{J^r,G^{re}\}\}+2if^{c8e}\{J^2,[G^{ke},\{J^r\}]\}-2if^{c8e}\{\{J^r,G^{re}\},[J^2,G^{k8}]\} \\
&+2if^{c8e}\{J^k,[\{J^i,G^{i8}\},\{J^r,G^{r8}\}]\}-4if^{c8e}[G^{ke},\{J^r,G^{r8}\}]+4if^{c8e}[G^{k8},\{J^r,G^{re}\}],
\end{aligned} \tag{B.46}$$

$$\begin{aligned}
&[G^{ia8},[\mathcal{D}_2^{i8},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{O}_3^{i8},G^{kc}]]\\
&+[\mathcal{O}_3^{i8},[\mathcal{D}_2^{i8},G^{kc}]]+[G^{i8},[\mathcal{O}_3^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{i8},[G^{i8},\mathcal{D}_2^{kc}]]\\
&=-2\{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\}-4\{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\}+d^{c8e}\{J^2,\{G^{ke},T^8\}\}\\
&+d^{c8e}\{\mathcal{D}_2^{k8},\{J^r,G^{re}\}\}+\frac{1}{2}d^{88e}\{J^2.\{G^{ke},T^c\}\}\\
&+\frac{1}{2}d^{88e}\{\mathcal{D}_2^{kc},\{J^r,G^{re}\}\}-if^{c8e}\{J^2,[G^{ke},\{J^r,G^{r8}\}]\}\\
&+2if^{c8e}\{J^2,[G^{k8},\{J^r,G^{re}\}]\}+if^{c8e}\{\{J^r,G^{re}\},[J^2,G^{k8}]\}\\
&-if^{c8e}\{\{J^r,G^{r8}\},[J^2,G^{ke}]\}-2if^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\}+9f^{c8e}f^{8eg}\mathcal{D}_2^{kg}\\
&-d^{ceg}d^{88e}\mathcal{D}_4^{kg}+d^{c8e}d^{8eg}\mathcal{D}_4^{kg}+\frac{11}{2}f^{c8e}f^{8eg}\mathcal{D}_2^{kg}+\frac{6}{N_f}\delta^{c8}\mathcal{D}_4^{k8},
\end{aligned} \tag{B.47}$$

$$[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]]=\frac{1}{2}\{\mathcal{D}_2^{kc},\{T^8,T^8\}\}-f^{c8e}f^{8eg}\mathcal{D}_2^{kg}+\frac{1}{2}f^{c8e}f^{8eg}\mathcal{D}_4^{kg}, \tag{B.48}$$

$$com9act=factor9.operatorbasis88 \tag{B.49}$$

$$com10act=factor10.operatorbasis88 \tag{B.50}$$

$$com10act=factor10.operatorbasis88 \tag{B.51}$$

$$com11act=factor11.operatorbasis88 \tag{B.52}$$

$$\begin{aligned}
[\mathcal{D}_2^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{D}_3^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_2^{kc}]]&=8\{D_2^{kc},\{T^8,T^8\},\{J^r,G^{r8}\}\}\\
&+4\{D_2^{k8},\{T^8,\{J^r,G^{rc}\}\}\}-10f^{c8e}f^{8eg}\mathcal{D}_3^{kg}+6f^{c8e}f^{8eg}\mathcal{D}_5^{kg},
\end{aligned} \tag{B.53}$$

$$\begin{aligned}
&[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]]=\\
&\frac{1}{2}\{J^2,\{G^{kc},\{T^8,T^8\}\}\}+\{J^2,\{G^{k8},\{T^c,T^8\}\}\}-4\{\mathcal{D}_2^{kc},\{T^8,\{J^r,G^{r8}\}\}\}\\
&-2\{\mathcal{D}_2^{k8},\{T^8,\{J^r,G^{rc}\}\}\}-\frac{1}{2}\epsilon^{kim}f^{c8e}\{J^2,\{T^e,\{J^i,G^{m8}\}\}\}+4f^{c8e}f^{8eg}\mathcal{D}_3^{kg}\\
&-\frac{9}{4}f^{c8e}f^{8eg}\mathcal{D}_5^{kg}-2f^{c8e}f^{8eg}\mathcal{O}_3^{kg}+\frac{1}{2}f^{8eg}f^{8eg}\mathcal{O}_5^{kg}
\end{aligned} \tag{B.54}$$

$$com15act=factor15.operatorbasis88 \tag{B.55}$$

$$com16act = factor16.operatorbasis88 \tag{B.56}$$

$$com17act = factor17.operatorbasis88 \tag{B.57}$$

$$com18act = factor18.operatorbasis88 \tag{B.58}$$

$$com19act = factor19.operatorbasis88 \tag{B.59}$$

$$com20act = factor20.operatorbasis88. \tag{B.60}$$

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